The transition from radiation dominated era to matter dominated era

After BBN, the universe was filled with matter and radiation. The radiation energy density dominates that of the mass, and we are in the **radiation dominated era**. But radiation density drops faster than mass density as the expansion continues.

Mass energy density:

$$U_{matter} = \frac{Mass \times c^2}{Volume}$$

$$U_{matter} \sim R^{-3}$$

Radiation energy density:

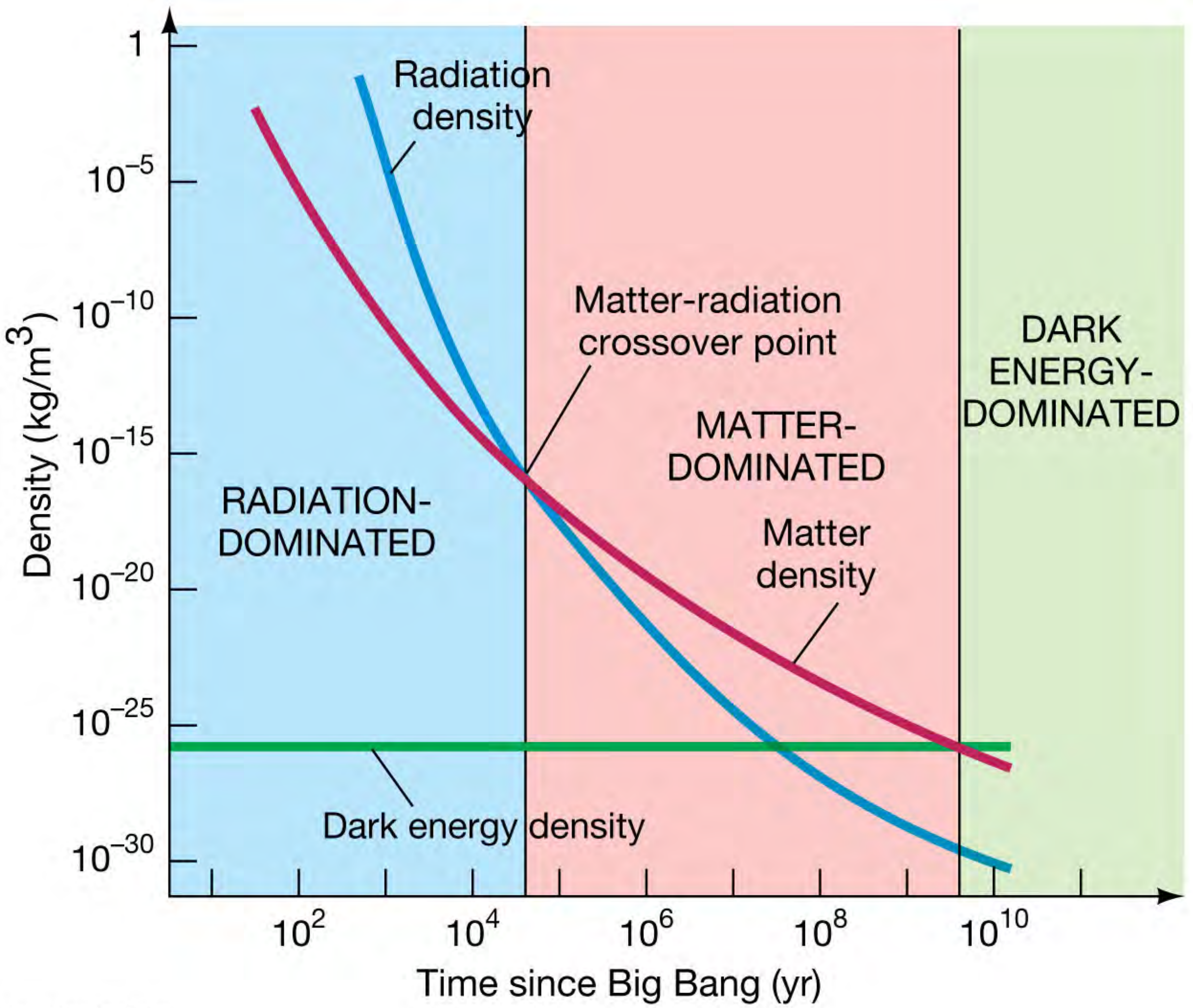
$$U_{rad} = \frac{N_{photon} \times E_{photon}}{Volume}$$

Photon energies drop as they get redshifted!

$$U_{rad} \sim \frac{R^{-1}}{R^3} \sim R^{-4}$$

At $t \approx 55{,}000$ years ($T \approx 9000$ K), radiation energy density drops below mass energy density, and we enter the **matter-dominated era**.

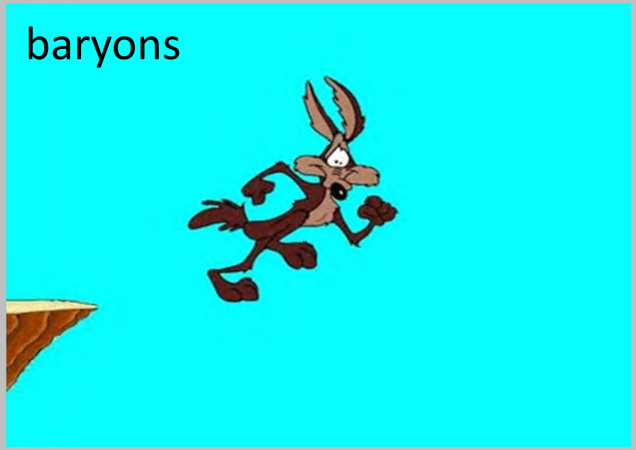
Time for gravity to do its thing!



Matter in the early universe

Two forms of matter:
- **Baryonic** (normal matter: protons, neutrons; including electrons), 10% – 15% of total mass
- **Non-baryonic dark matter** (????), 85% − 90% of total mass

Focus now on baryonic matter.



Before recombination ($t \lesssim 350,000$ years):

Photons scatter off of free electrons (Thompson scattering), providing photon pressure which prevents baryons from falling into gravitational potential wells. They are "suspended".

After recombination ($t > 350,000$ years)

Free electrons combine with free protons to form hydrogen atoms, no more Thompson scattering, no more photon pressure. Baryons can start falling into potential wells.

Matter in the early universe

Two forms of matter:

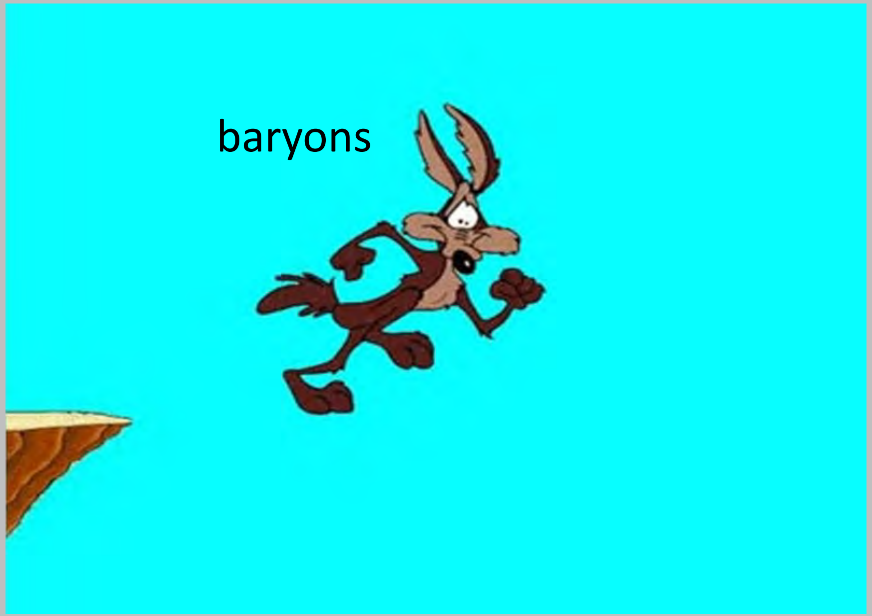
- **Baryonic** (normal matter: protons, neutrons; including electrons), 10% – 15% of total mass
- **Non-baryonic dark matter** (????), 85% − 90% of total mass

In the early universe ($t \lesssim 350{,}000$ years) the Universe is too hot for bound hydrogen to form, so all the baryonic matter is ionized: free protons and electrons, with some helium nuclei and other leftovers from BBN. All that ionized baryonic matter is mixed with photons and dark matter.

Remember: photons and electrons easily scatter off one another (Thompson scattering), which is why the early universe is opaque: Light cannot free stream.

But that also means that photon pressure keeps the baryonic matter from gravitationally contracting.

Imagine an overdense lump of the universe at this time. The excess gravity is wanting to pull more mass inwards: gravitational contraction. But photon pressure keeps the electrons keep the from falling inwards: they stay "suspended" (along with the protons, which are electrostatically coupled to the electrons).



Recombination

At $t \approx 350{,}000$ years (redshift $z \approx 1000$), the temperature drops below $\mathrm{T} \approx 3{,}000$ K which is cool enough for protons and electrons to combine to form bound hydrogen atoms.

No more free electrons, no Thompson scattering of photons. The universe becomes transparent, the photon pressure goes away, and suddenly baryonic matter can start to collapse under gravity.

The Growth of structure

Because matter and radiation are coupled before recombination, the temperature fluctuations in the cosmic microwave background are related to the baryonic density fluctuations at recombination: at $z \approx 1000$, $\Delta T/T \approx \Delta\rho/\rho \approx 10^{-5}$.

Hubble (and now JWST) also detect lots of galaxies forming by $z \approx 5$. A galaxy is a very strong overdensity: $\Delta\rho/\rho \approx 10^{5}$.

This is orders of magnitude of growth in density in less than a billion years. *Can gravity work that fast?*

The Growth of structure

Because matter and radiation are coupled before recombination, the temperature fluctuations in the cosmic microwave background are related to the baryonic density fluctuations at recombination: at $z \approx 1000$, $\Delta T/T \approx \Delta\rho/\rho \approx 10^{-5}$.

Hubble (and now JWST) also detect lots of galaxies forming by $z \approx 5$. A galaxy is a very strong overdensity: $\Delta\rho/\rho \approx 10^{5}$.

This is orders of magnitude of growth in density in less than a billion years. *Can gravity work that fast?*

Using the Friedman Equation to study structure formation

Remember, at early times the density is high enough that matter dominates over lambda. And also, observations told us that universe is spatially flat and stays flat. So to study the universe at these times, we can use the Friedman equation for a flat ($k = 0$) matter dominated ($\Lambda = 0$) universe.

$$\left(\frac{\dot{R}}{R}\right)^2 - \frac{8}{3}\pi G\rho - \frac{1}{3}\Lambda c^2 = -\frac{kc^2}{R^2}$$

$$H^2 - \frac{8}{3}\pi G\rho = 0$$

Hubble **parameter** – changes with time. Not Hubble constant (H_0)

Evolution of density fluctuations

average density of universe

To describe the Universe as a whole, start with the Friedman equation for a flat ($k = 0$) matter dominated ($\Lambda = 0$) universe.

$$H^2 - \frac{8}{3}\pi G\bar{\rho} = 0$$

Now consider a small piece of the universe (“a bubble”) which has a higher-than-average density: an overdensity. In that region, space is not flat, because it has more matter. So it gets its own Friedmann equation:

$$H^2 - \frac{8}{3}\pi G\rho' = -\frac{k}{R^2}$$

excess density of bubble

Now subtract the first equation from the second to get

$$-\frac{8}{3}\pi G(\rho' - \bar{\rho}) = -\frac{k}{R^2}$$

or, collecting terms:

$$(\rho' - \bar{\rho}) = -\frac{3k}{8\pi G R^2}$$

Evolution of density fluctuations (continued)

Now lets define a quantity δ which is the **fractional overdensity** of the bubble. $\delta = 0$ is average density (i.e., no overdensity), $\delta = -1$ is absolute emptiness (a strong underdensity), while $\delta \gg 1$ is a strong overdensity.

excesss density of bubble

average density of universe

$$\delta \equiv \left(\frac{\rho' - \bar{\rho}}{\bar{\rho}}\right)$$

Putting that into our equation for the difference between ρ' and $\bar{\rho}$

$$(\rho' - \bar{\rho}) = -\frac{3k}{8\pi G R^2}$$

We get

$$\delta = -\frac{3k}{8\pi G \bar{\rho} R^2}$$

And if we only pay attention to evolving quantities (R and ρ) we can think about how δ evolves:

$$\delta \sim \frac{1}{\bar{\rho} R^2} \sim \frac{1}{R^{-3} R^2} \sim R$$

So the fluctuation grows as R grows. But we also know that scale factor and redshift are related by $R = 1/(1+z)$, so rewrite this in terms of redshift:

$$\delta \sim (1+z)^{-1}$$

Evolution of density fluctuations (continued)

So if $\delta \sim (1+z)^{-1}$, we can relate the density fluctuations at two different times ("initial" and "final") by the expression:

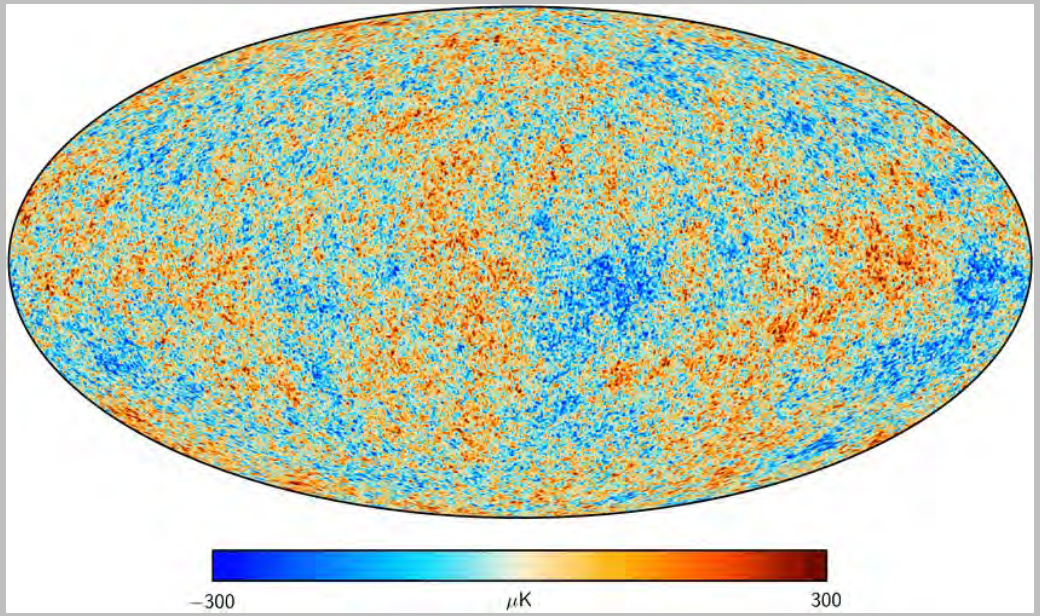
$$\frac{\delta_f}{\delta_i} = \frac{(1+z)_i}{(1+z)_f}$$

If we start with $\delta_i \approx 10^{-5}$ at $z \approx 1000$, by a redshift of $z \approx 5$, the overdensity should have grown to be:

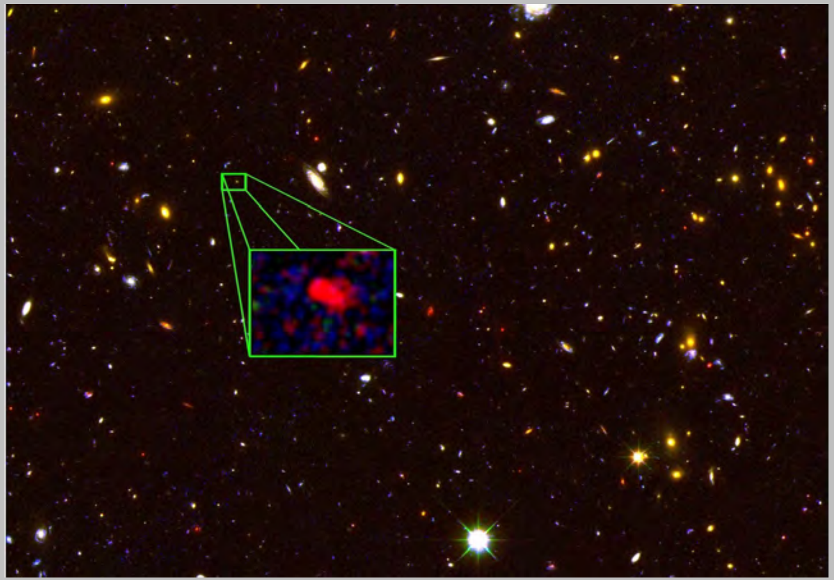
$$\delta_f = \delta_i \frac{(1+z)_i}{(1+z)_f} = 10^{-5}\left(\frac{1001}{6}\right) \approx 0.002$$

Oops! That's off by many orders of magnitude. By themselves, these fluctuations in baryonic mass ***aren't strong enough*** to grow into galaxies and galaxy clusters we see at high redshift.

What have we forgotten? ***Dark matter***.



Initial values: $\delta_i \approx 10^{-5}$ at $z_i \approx 1000$



Final values: $\delta_i \approx 10^{5}$ at $z_f \approx 5$

Dark Matter and Structure Formation

Non-baryonic dark matter does not interact with other particles or photons in any way but through gravity.

Dark matter overdensities could grow freely well before recombination, and there's more dark matter than baryonic matter.

So strong gravitational potential wells were already in place for the baryons to collapse into once recombination occurs.

Once an overdense region gets to $\delta > 1$, its dense enough to govern its own growth, and it decouples from the overall expansion of the Universe. It gravitational collapse happens roughly on a free-fall timescale:

$$t_{ff} \approx \sqrt{1/G\rho}$$

Object	Density (ρ) $M_{\odot}\, pc^{-3}$	Overdensity (δ)
Globular Cluster	100	10^9
Galaxy	10^{-3}	10^4
Galaxy Cluster	10^{-5}	100

Note: Low mass things have higher densities and are the first to form! Massive things form later.

Flavors of Dark Matter

Baryonic Dark Matter

Examples: Faint brown dwarfs, planets, diffuse gas clouds, free-floating space donkeys

Two fatal problems we have already discussed:

- Can’t form structure fast enough because they can’t grow until after recombination.

- Ruled out by big bang nucleosynthesis arguments ($\Omega_b \ll \Omega_m$)

Baryonic dark matter models do not work.

Non-baryonic dark matter

Classified by the characteristic random velocities (energies):

- **Hot dark matter**: particles moving at relativistic speeds
- **Cold dark matter**: particles moving at much slower speeds

Flavors of Dark Matter

Hot Dark Matter: Particles moving at high relativistic speeds
Example: neutrino

In the early universe, HDM particles moving at relativistic speeds will quickly escape from low mass density fluctuations. These fluctuations will no longer be bound, and will not collapse. The only fluctuations that survive are things with masses $\gtrsim 10^{15}\ M_{\odot}$ (massive galaxy cluster scales). These take a long time to collapse, since they are low density.

Once they collapse, and the density increases, smaller structures can start to collapse inside them, a process called "fragmentation". (The way individual stars form inside a collapsing gas cloud)

So in Hot Dark Matter models:

- **Structure forms "top down"**: big things first, then smaller and smaller things.
- **Structure forms slowly**: Have to wait for the big low density things to collapse before structure can form.
- **Galaxies form late** in the Universe's history.

This is not what we see – hot dark matter models do not work!

Flavors of Dark Matter

Cold Dark Matter: Particles moving at low speeds
Example: NO KNOWN OBJECTS

In the early universe, CDM particles move much more slowly and can be bound into low mass "halos" of dark matter. Once recombination occurs, baryons collapse into these low mass halos first, then over time low mass halos (of dark matter and baryons) continue to merge on larger and larger size scales: Hierarchical formation.

So in Cold Dark Matter models:

- **Structure forms "bottom up"**: small things form first, then merge together over time to form bigger things.
- **Structure forms early**: As soon as recombination hits, structure can begin forming quickly
- **Galaxies form early, galaxy clusters form later**.

This is a much better model for what we see: galaxies forming in the early Universe, galaxy clusters growing at later times.

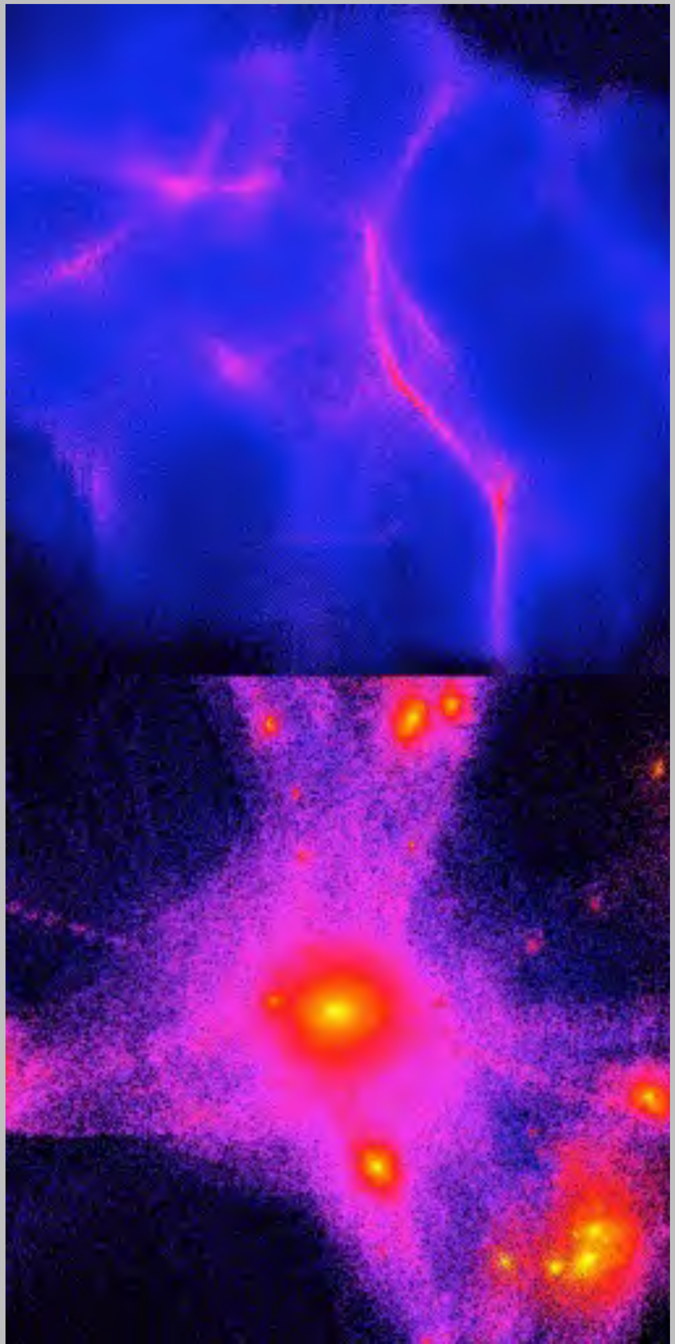
HDM: very little structure early;

CDM: much more early structure.

HDM: no dwarf galaxies at late times.

CDM: many more low mass galaxies at late times.

Hot Dark Matter



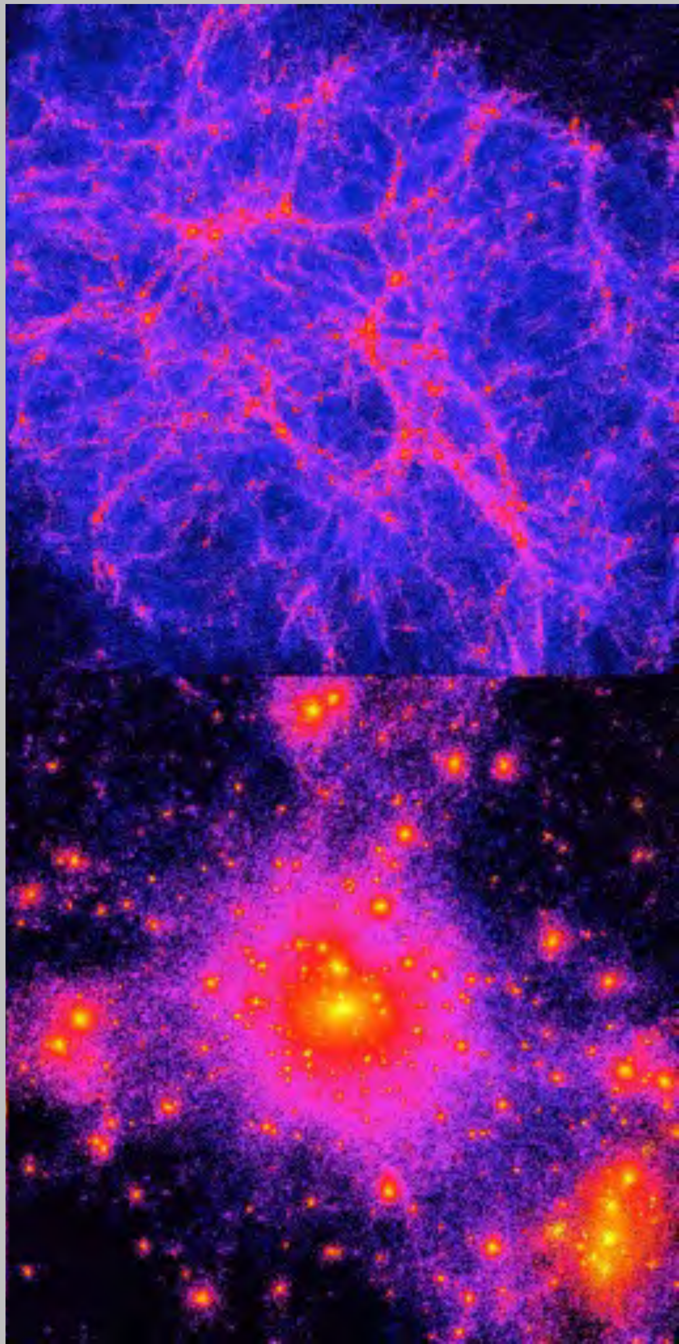
Early Universe

⟺

Today

⟺

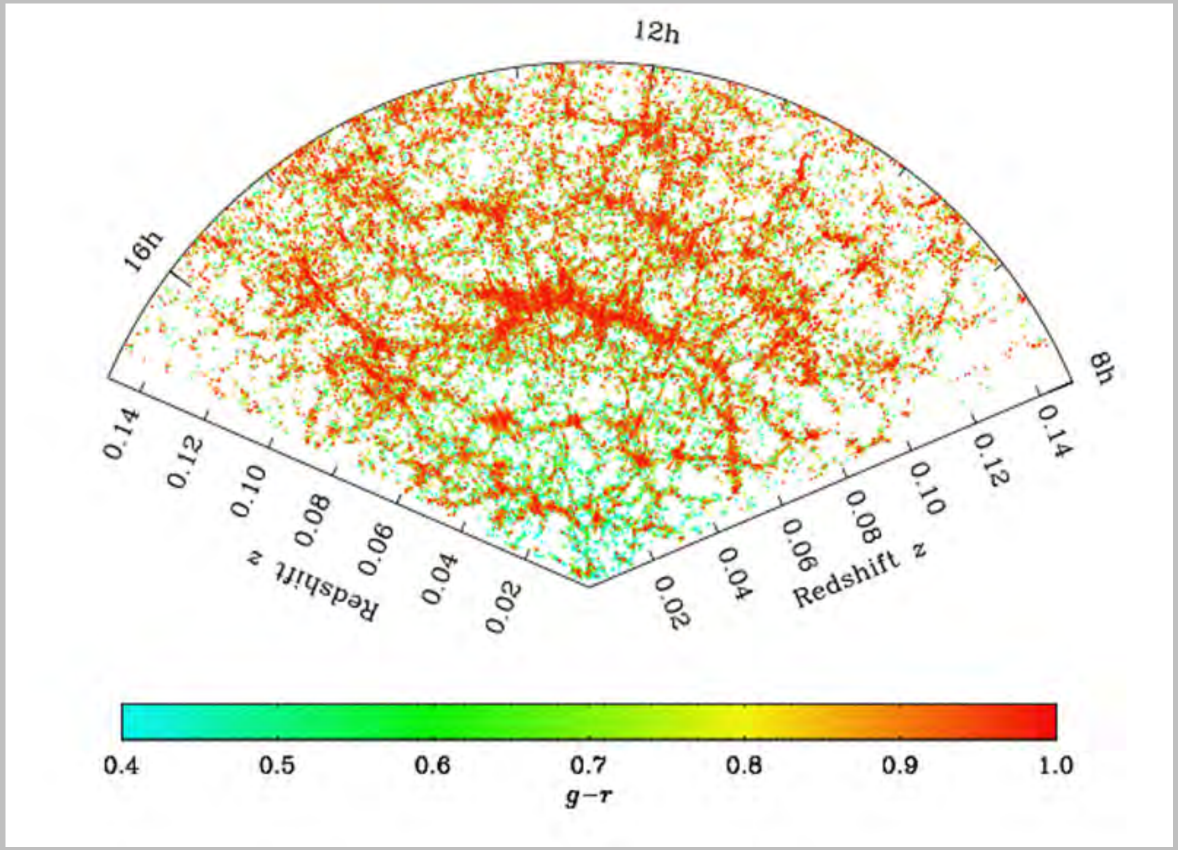
Cold Dark Matter



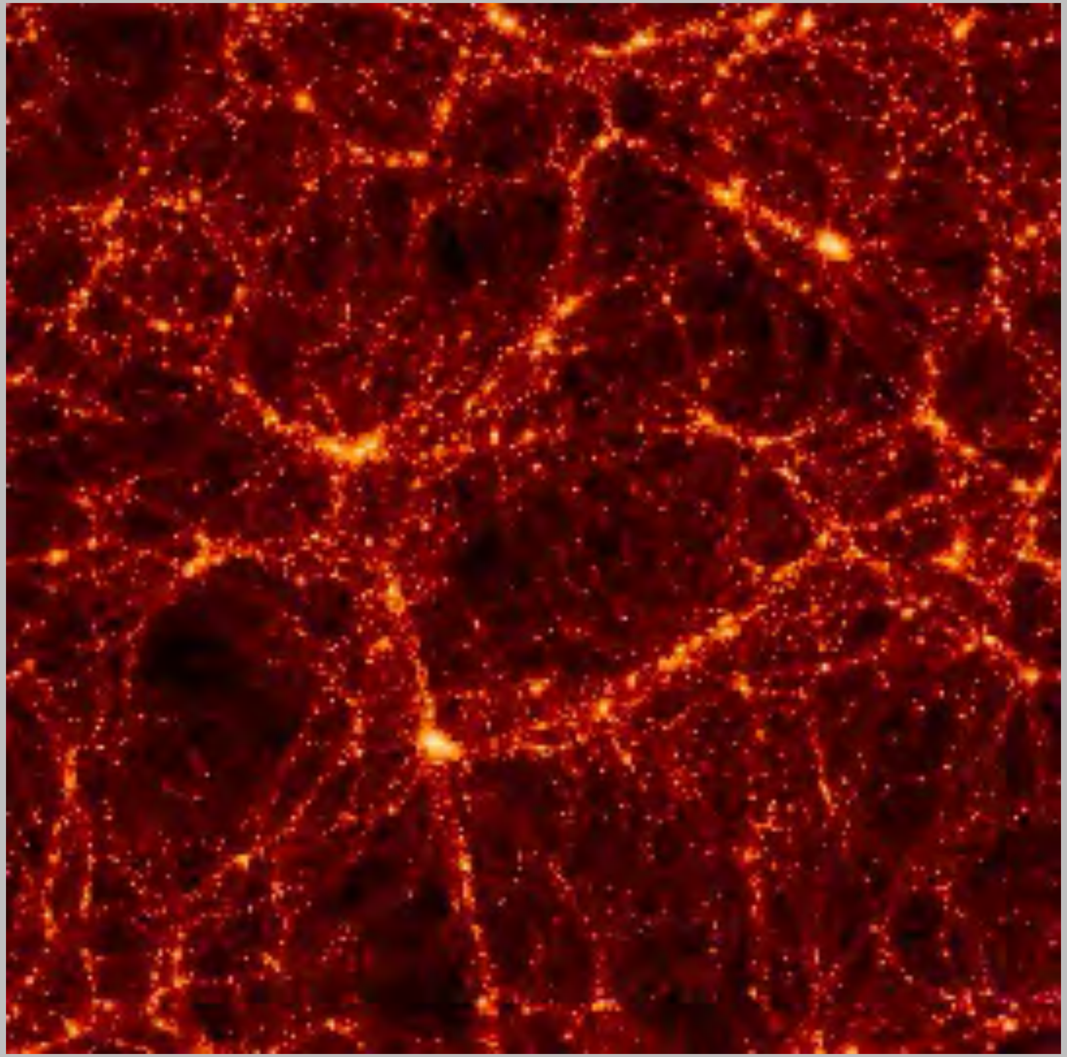
Simulations of structure formation
(courtesy ITC/Zurich)

Structure Formation under Cold Dark Matter models

Connecting observations of large scale structure.....



...to predictions from theoretical models



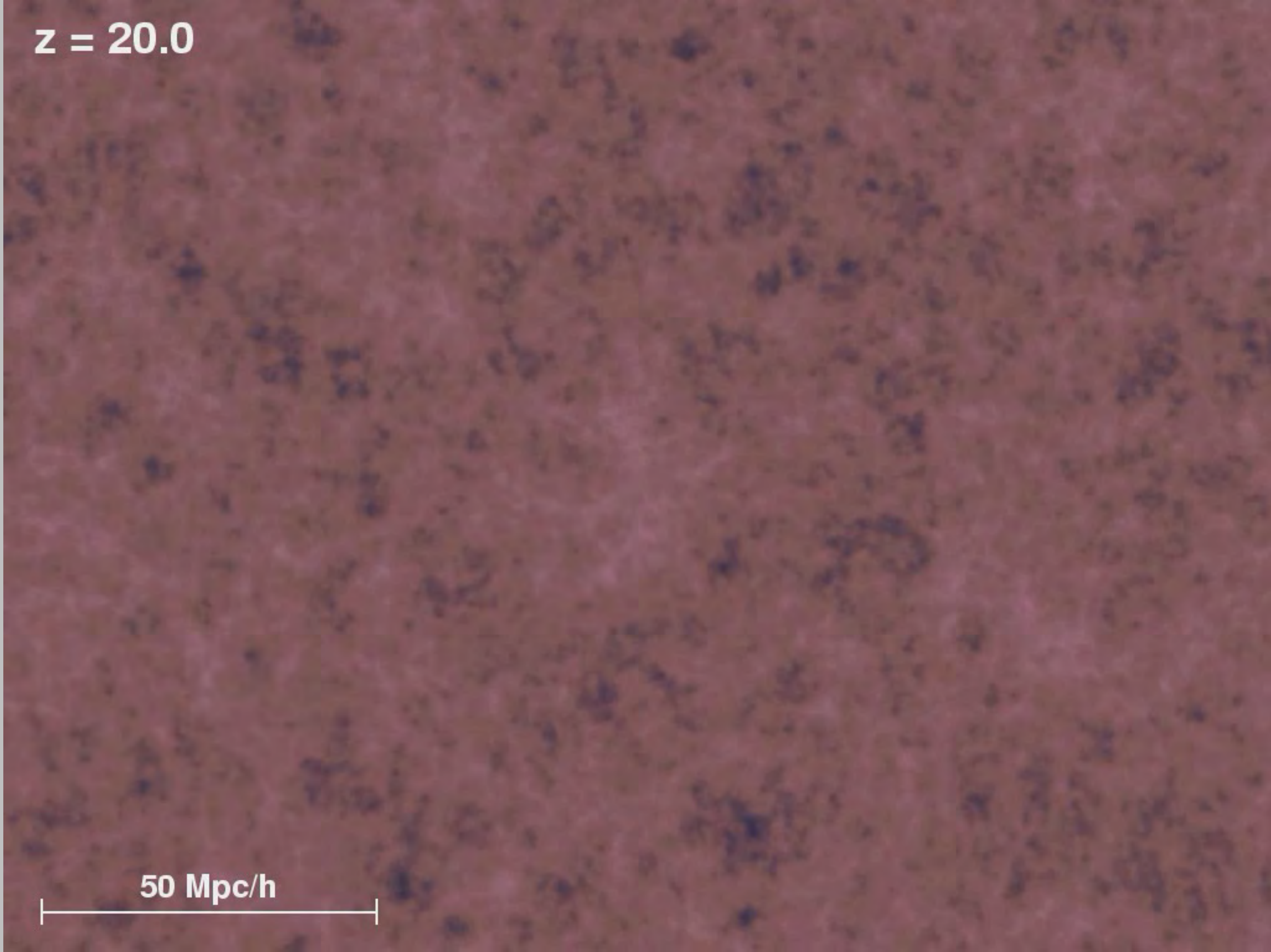
The formation of structure

Small fluctuations in the mass density at early time grow in strength due to gravity pulling mass together.

Mass forms filamentary structure, collects in dense regions at the intersections of filaments.

Large voids also grow as matter empties out of them into the filaments.

Dark matter simulation from the Millenium Simulation (Springel+05)

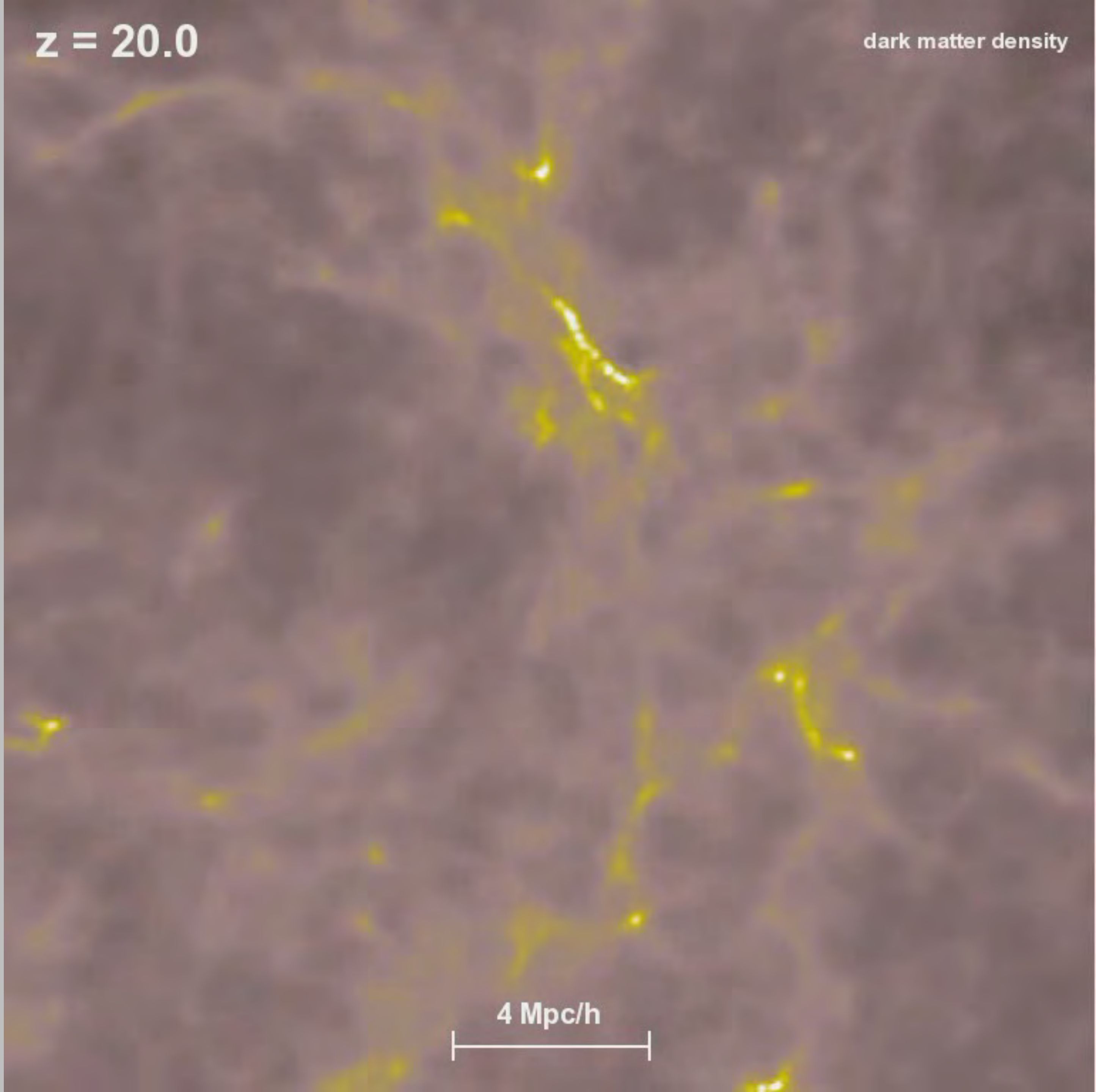


Forming Galaxy Clusters

In cold dark matter models, clusters form hierarchically: small lumps merge together to form bigger lumps, which merge to form even bigger lumps, etc.

Clusters grow over time, and the rate at which they grow depends on the density of the universe and its expansion history.

Dark matter simulation from the Millenium Simulation (Springel+05)

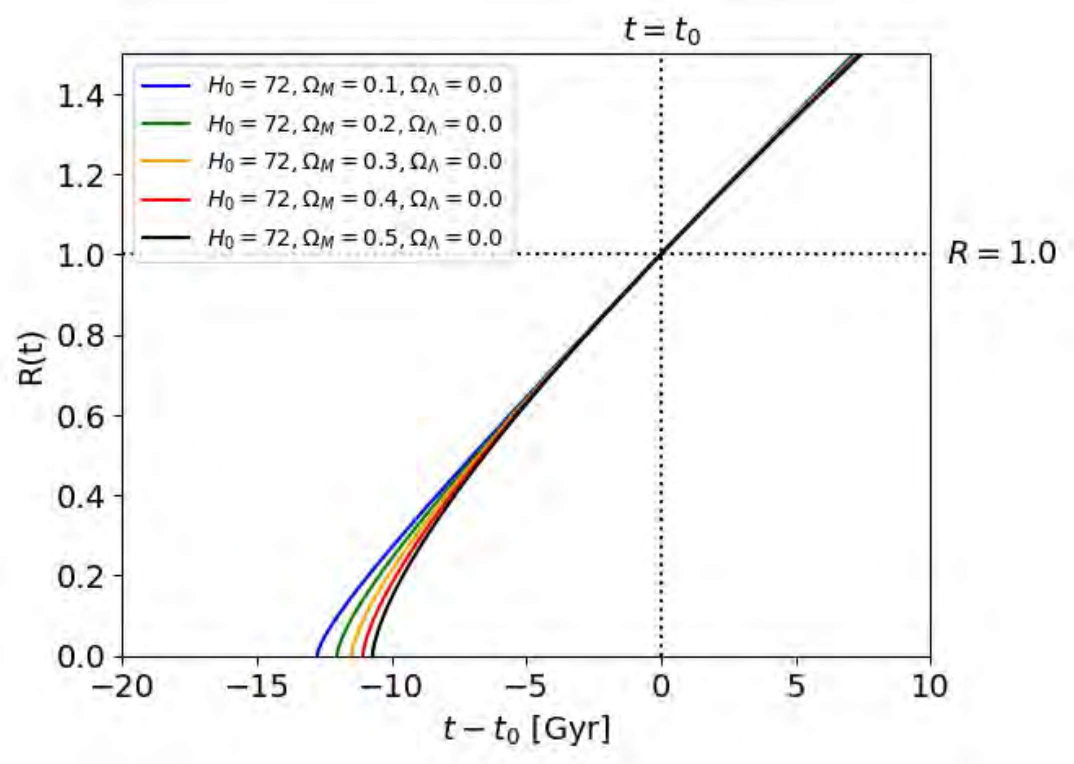


Structure Formation under Cold Dark Matter models

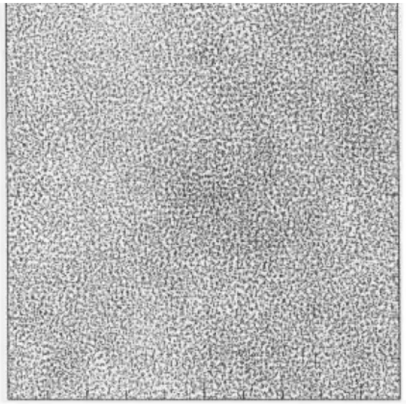
First, just consider Universes where there is no dark energy, so $\Omega_\Lambda = 0.0$.

The more mass there is (i.e., bigger Ω_m) the more structure there is at present day. More mass ⇒ stronger gravity ⇒ structure grows faster.

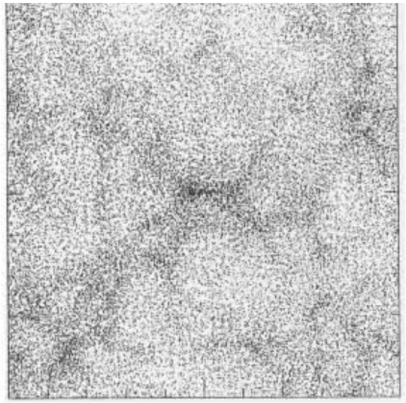
Structure seems "right" at values of $\Omega_m \approx 0.4$ or so.
But to get the age right, we want $\Omega_m \lesssim 0.2$. *That's a problem!*



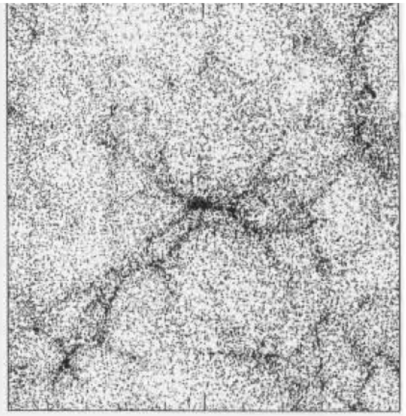
$\Omega_m = 0.1$



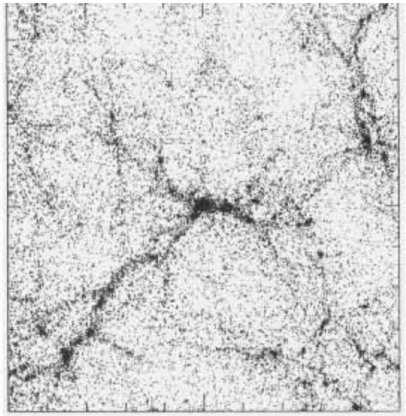
$\Omega_m = 0.2$



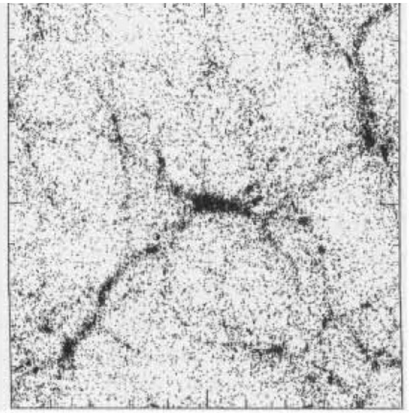
$\Omega_m = 0.3$



$\Omega_m = 0.4$



$\Omega_m = 0.5$



Simulated present-day universes.

All models have $\Omega_\Lambda = 0.0$

Cole+97

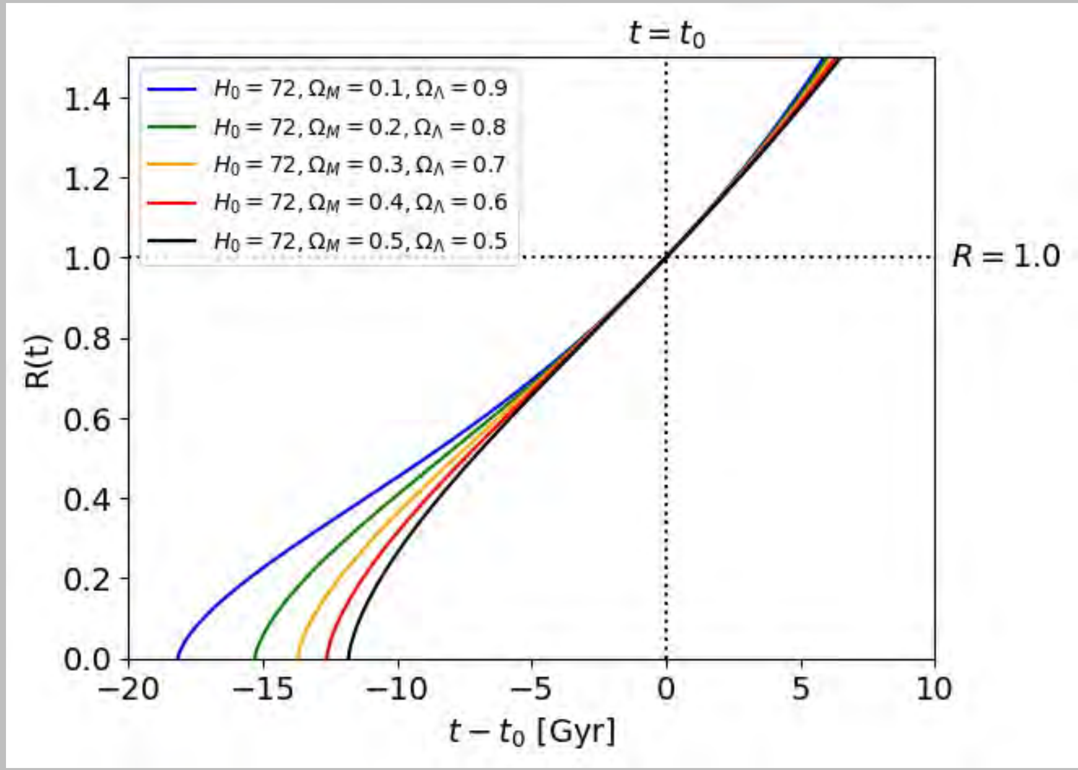
Structure Formation under Cold Dark Matter models

Now, just consider flat Universes where $\Omega_m + \Omega_\Lambda = 1.0$.

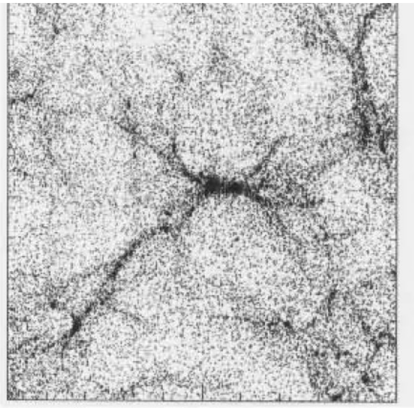
At fixed Ω_m, universes have much more structure than before (when we held $\Omega_\Lambda = 0$).

Dark energy makes universes older, so more time for structure to grow.

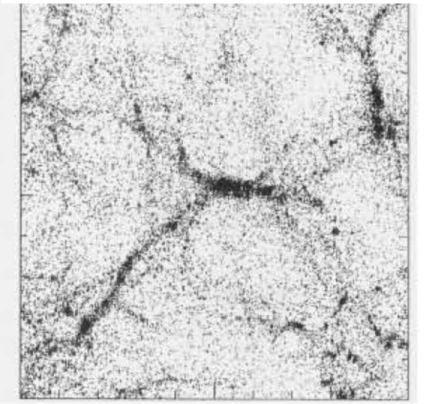
Get a good match to structure *and* age at $\Omega_m = 0.3, \Omega_\Lambda = 0.7$



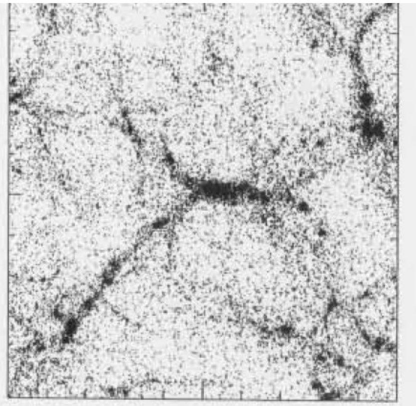
$\Omega_m = 0.1\ \Omega_\Lambda = 0.9$



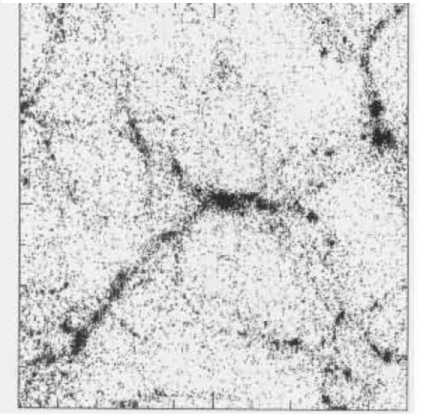
$\Omega_m = 0.2, \Omega_\Lambda = 0.8$



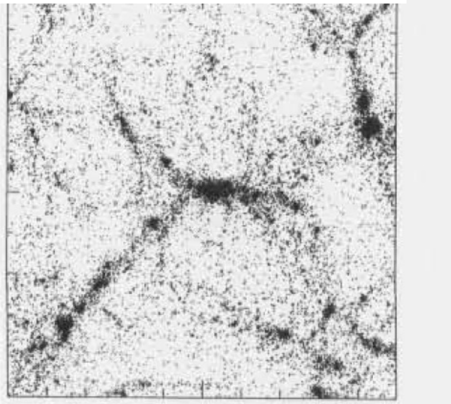
$\Omega_m = 0.3, \Omega_\Lambda = 0.7$



$\Omega_m = 0.4, \Omega_\Lambda = 0.6$



$\Omega_m = 0.5, \Omega_\Lambda = 0.5$



Simulated present-day universes.

All models have $\Omega_m + \Omega_\Lambda = 1.0$

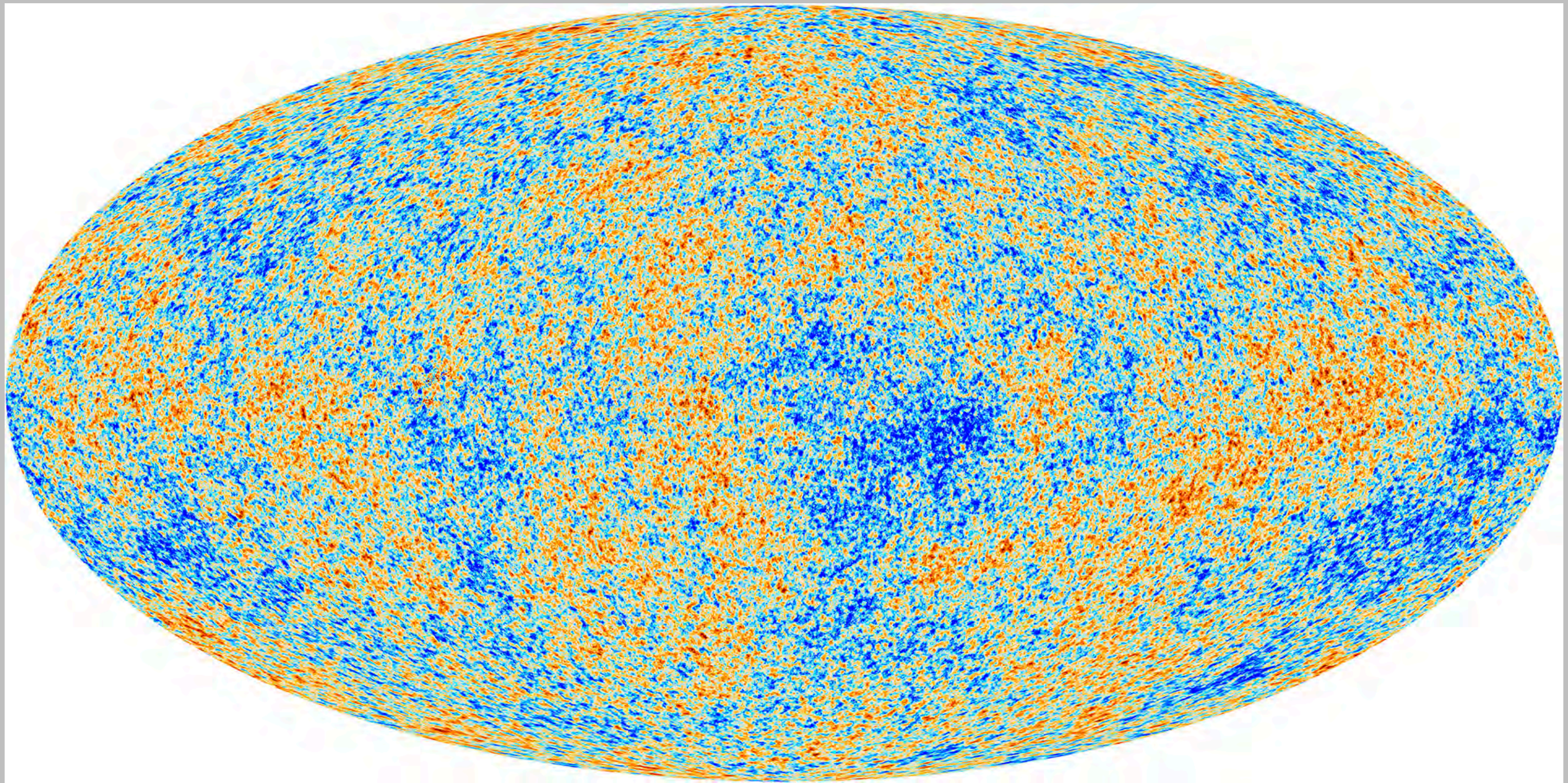
Cole+97

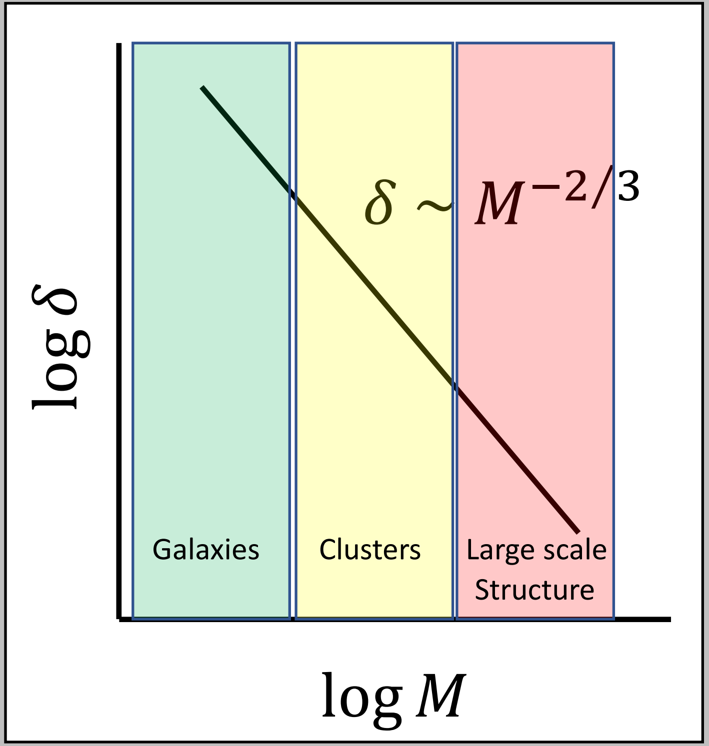
Growing Galaxies

Remember that the fluctuations in mass density traced by the microwave background have characteristics sizes of about 65 Mpc -- much larger than galaxies and galaxy clusters. On smaller scales inside those fluctuations (and unresolved by current CMB data) are the fluctuations destined to grow into galaxies.

Fluctuations on smaller mass scales are stronger overdensities ($\delta \equiv \Delta\rho/\rho$). $\Rightarrow$

Low mass things form first, then merge to form large things: ***hierarchical assembly***.





Fluctuation Power Spectrum: More "power" (stronger overdensities) on smaller mass scales.

Hierarchical Growth of Galaxies and Merger Trees

Merger Tree:

- Time runs down the chart
- Width of branches/trunk = galaxy masses
- Branches meeting = galaxies merging.

Early times: many seperate low mass (and gas-rich) galaxies at high redshift.

As time goes by: ongoing mergers, star formation

Late times: Large galaxy in local universe

Questions:

- *When did this galaxy form?*
- *When did this galaxy's stars form?*
- *What do we mean by the age of a galaxy?*

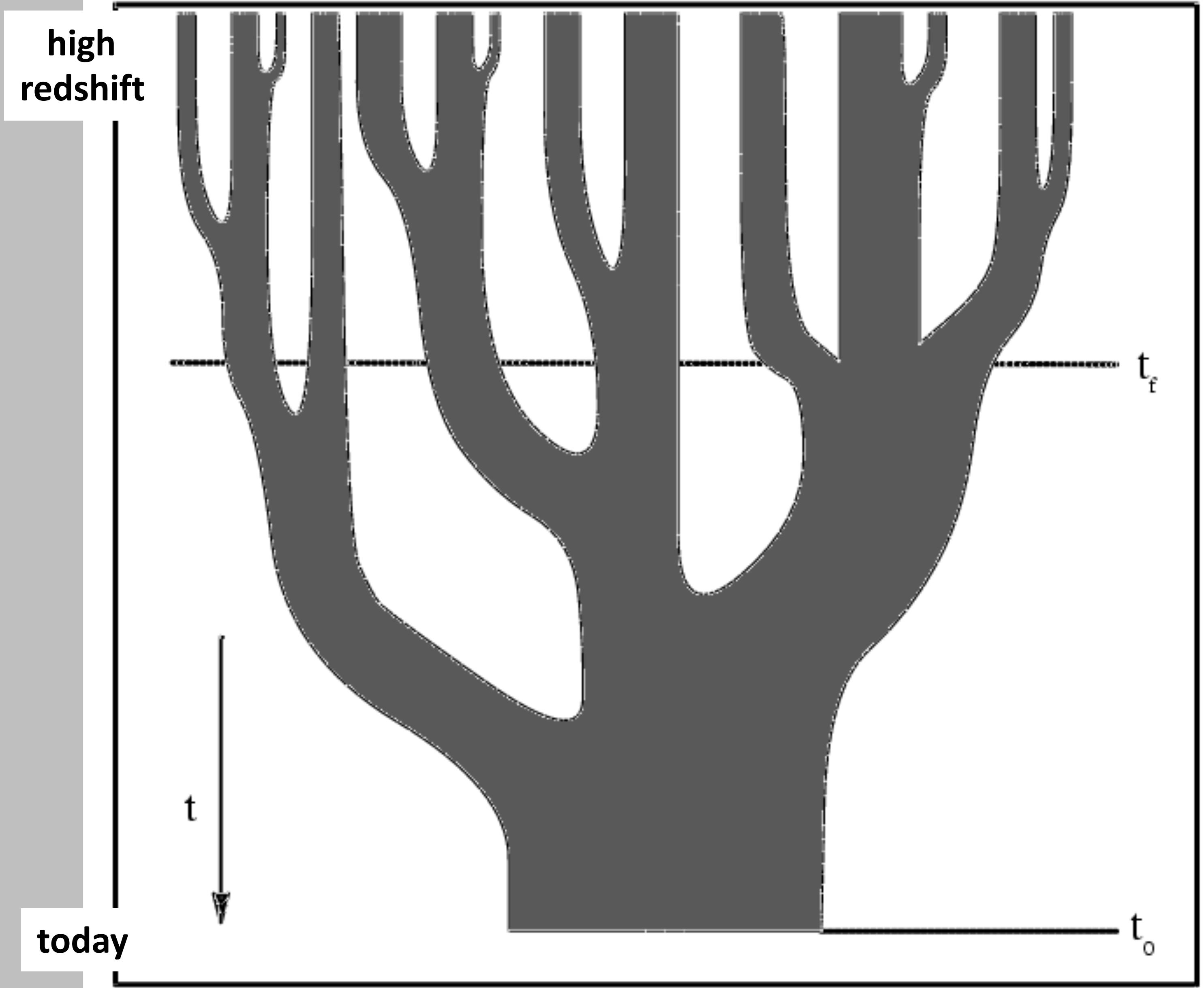


Figure 6. A schematic representation of a "merger tree" depicting the growth of a halo as the result of a series of mergers. Time increases from top to bottom in this figure and the widths of the branches of the tree represent the masses of the individual parent halos. Slicing through the tree horizontally gives the distribution of masses in the parent halos at a given time. The present time t_0 and the formation time t_f are marked by horizontal lines, where the formation time is defined as the time at which a parent halo containing in excess of half of the mass of the final halo was first created.

Dark Matter Halo mass function

Plot the (log) number (N) of dark matter halos of a given mass (M) as a function of redshift (z).

As low mass galaxies merge together to form higher mass halos, massive halos become more common.

Predictions:

- massive galaxies should be rare in the very early universe ($z > 5$).
- galaxy clusters should be rare at $z > 1$ (first half of Universe's history).