Cosmological Parameters

1. The **Hubble Constant** (normalized expansion rate today):

$$H_0 = \left(\dot{R}/R\right)_{t=t_0} \approx 68 - 72 \text{ km/s/Mpc}$$

More generally, the *Hubble Parameter* (changes with time)

$$H \equiv \dot{R}/R$$

2. The **Matter Density parameter** (normalized mass density):

$$\Omega_m = \frac{\rho}{\rho_{crit}}, \qquad \Omega_{m,0} \approx 0.3$$

3. The **Dark Energy Density parameter** (normalized dark energy density):

$$\Omega_\Lambda = \frac{\Lambda}{\Lambda_{crit}}, \qquad \Omega_{\Lambda,0} \approx 0.7$$

The Dynamics Equation

$$\frac{\ddot{R}}{R} = -\frac{4\pi}{3}G\rho + \frac{1}{3}\Lambda c^2$$

The Friedman Equation

$$\left(\frac{\dot{R}}{R}\right)^2 - \frac{8}{3}\pi G\rho - \frac{1}{3}\Lambda c^2 = -\frac{kc^2}{R^2}$$

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The R(t) plot: Understanding the parameters graphically and intuitively

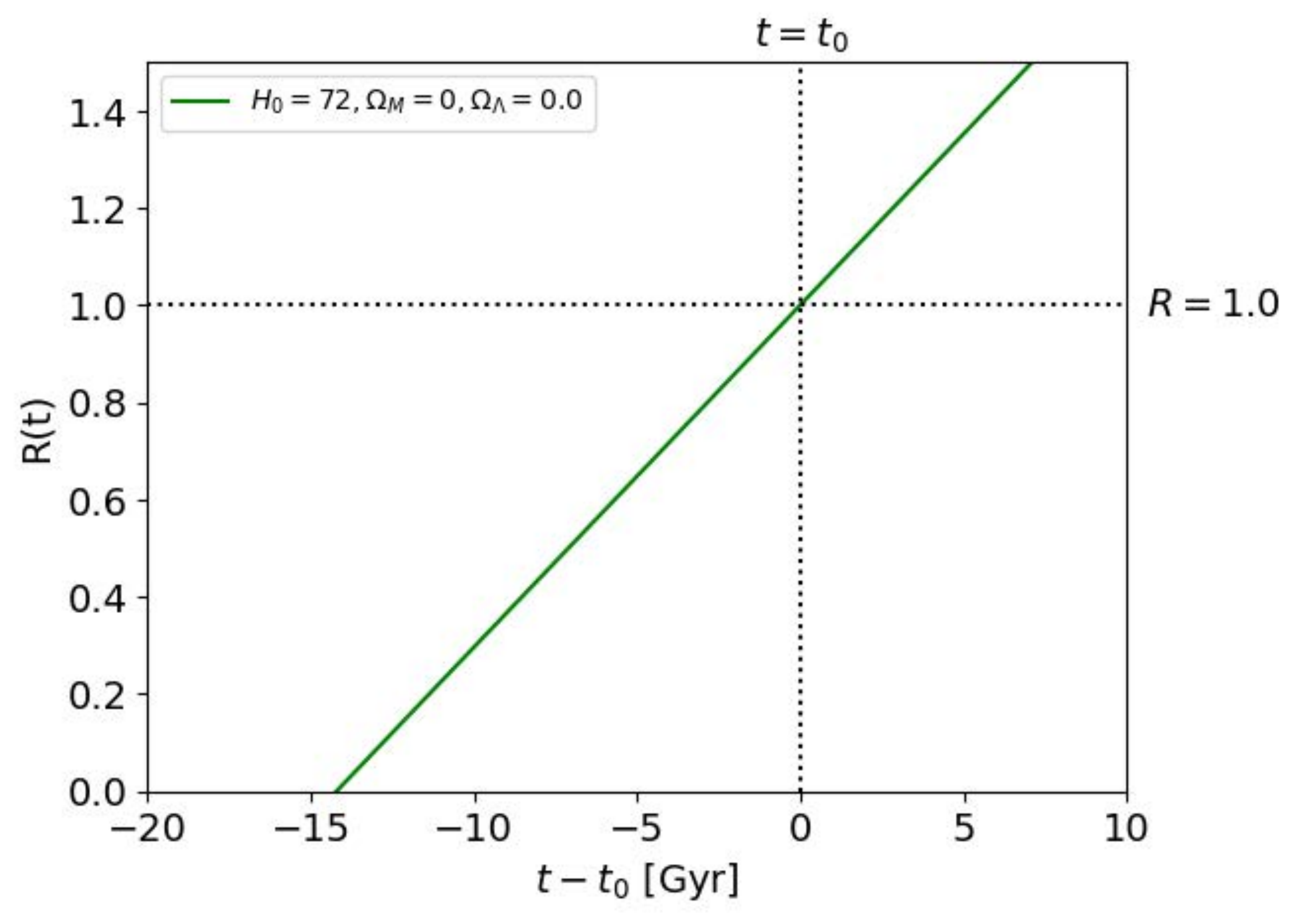
Imagine a Universe that is expanding at a constant rate (Homer Simpson Universe).

In this Universe, R(t) is a straight line, and Homer worked out an age of 13.9 billion years for $H_0 = 72$ km/s/Mpc.

The Hubble Parameter is given by

$$H \equiv \frac{\dot{R}}{R}$$

so it's the slope of the R(t) line divided by the scale factor itself (R).



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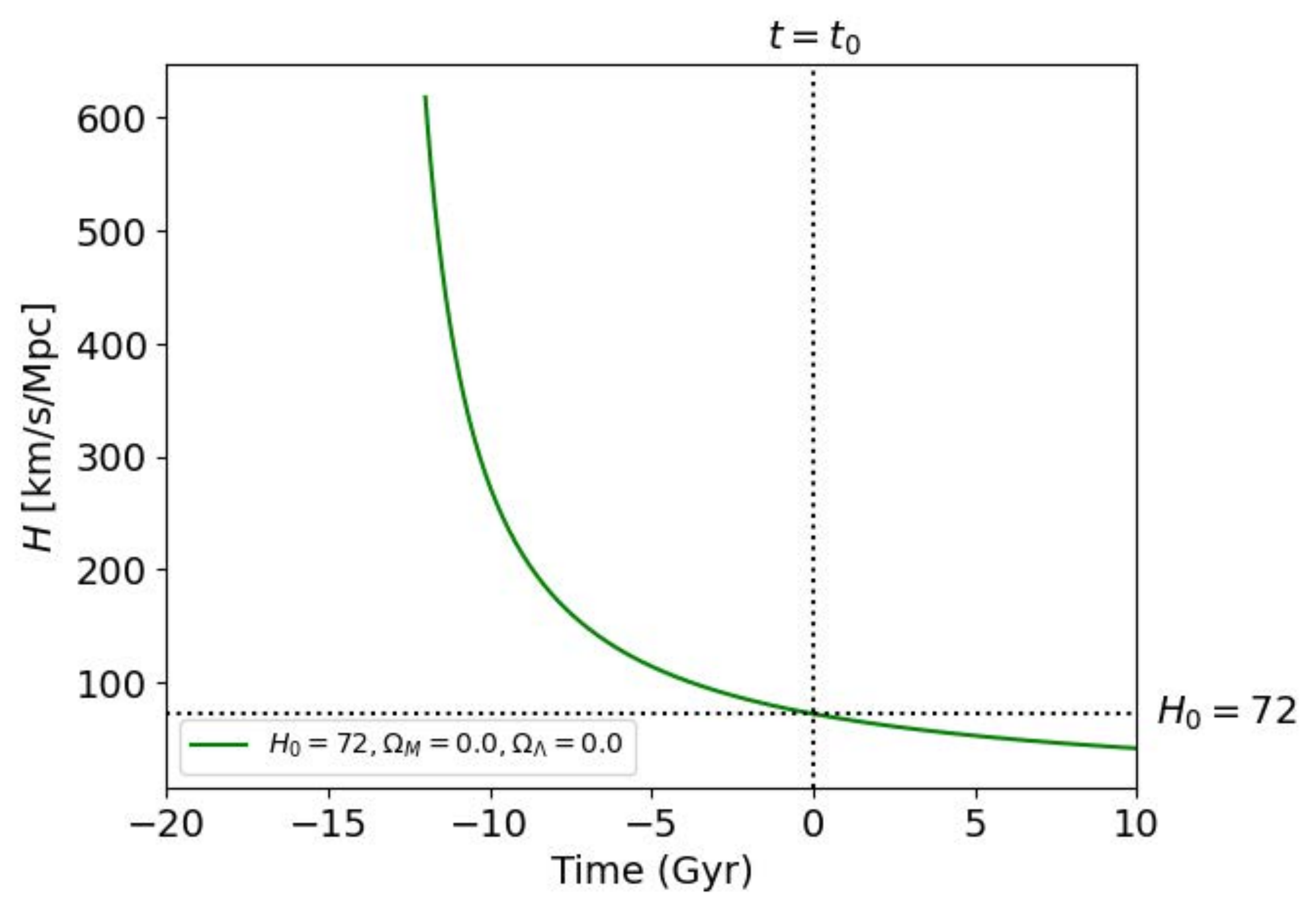

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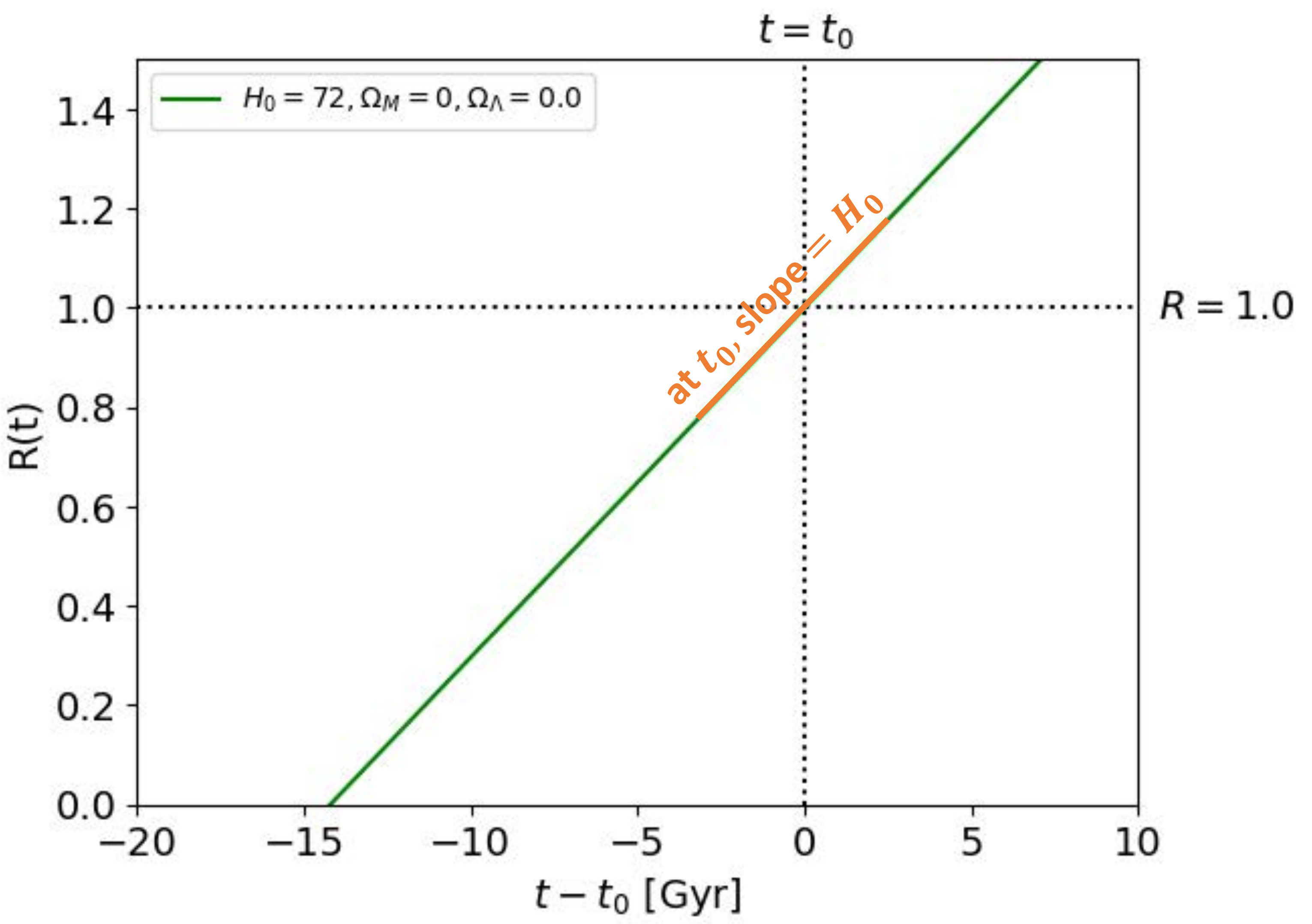
The Hubble Constant is the Hubble Parameter **today** (at $t = t_0$).

So it is the slope of the line today, divided by $R_0 = 1$.

$$H_0 \equiv \frac{\dot{R}(t_0)}{R_0} = \dot{R}(t_0)$$

So the Hubble Constant is *essentially* just the slope of the line today.



The R(t) plot: Understanding the parameters graphically and intuitively

Imagine a Universe that is expanding at a constant rate (Homer Simpson Universe).

Changing the Hubble Constant means changing the slope at $t = t_0$.

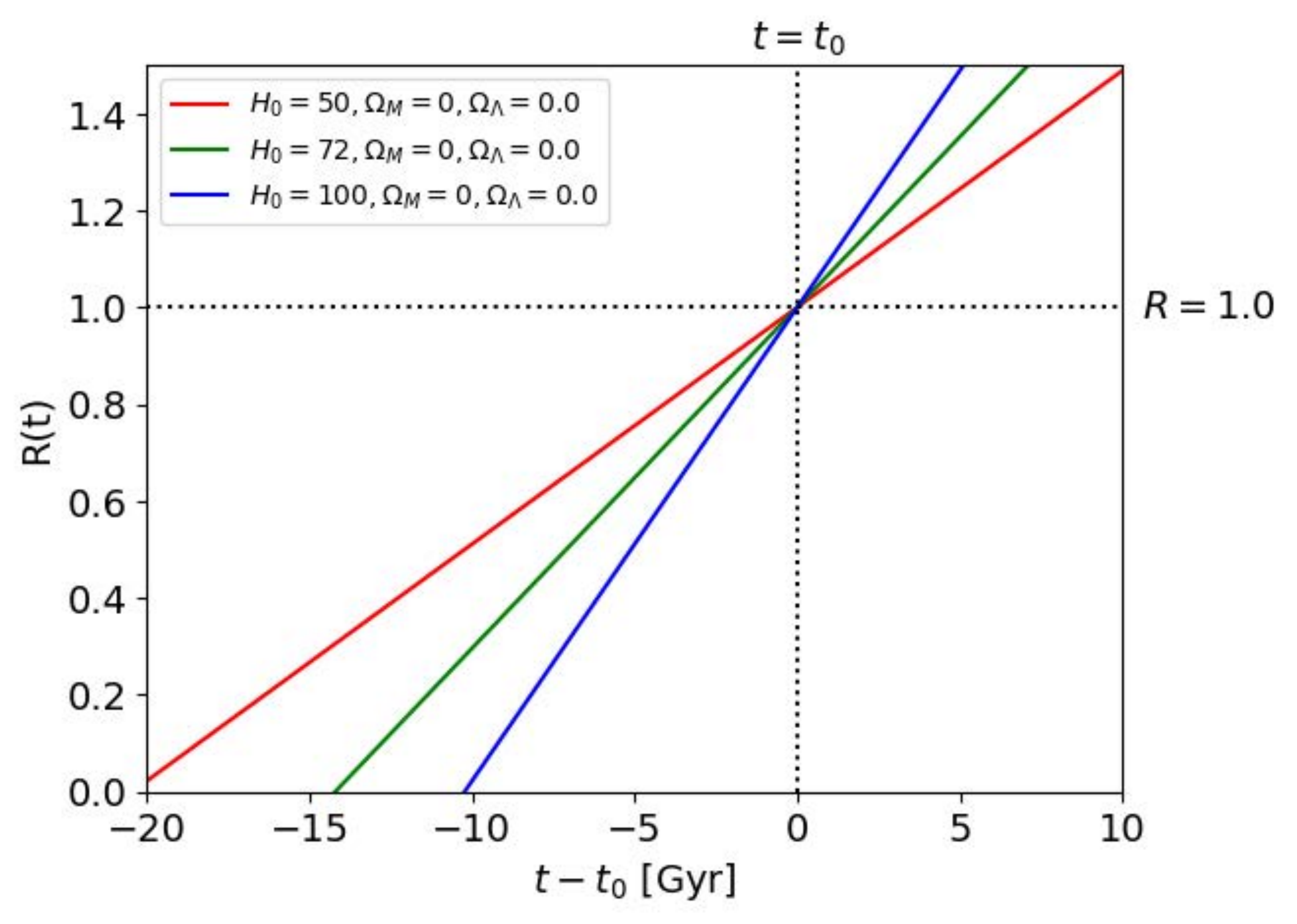
Remember the boundary conditions! No matter how you change the parameters of the Universe, R(t) has to satisfy the following conditions:

- $R(t_0) = 1$
- $H(t_0) = \left(\frac{\dot{R}(t_0)}{R_0}\right) = \dot{R}(t_0) = H_0$

This changes t_0, the age of the Universe! This is why, for a contant expansion universe,

$$t_0 = 1/H_0$$

A higher Hubble constant results in a younger Universe, and vice-versa.



The R(t) plot: Understanding the parameters graphically and intuitively

Now add matter to the Universe (so $\Omega_m > 0$) but no cosmological constant ($\Omega_\Lambda = 0$). Matter has gravity, and gravity slows down the expansion: deceleration. The Universe must have been expanding faster in the past, so R(t) must be "bending downwards."

But we still have to obey the boundary conditions!

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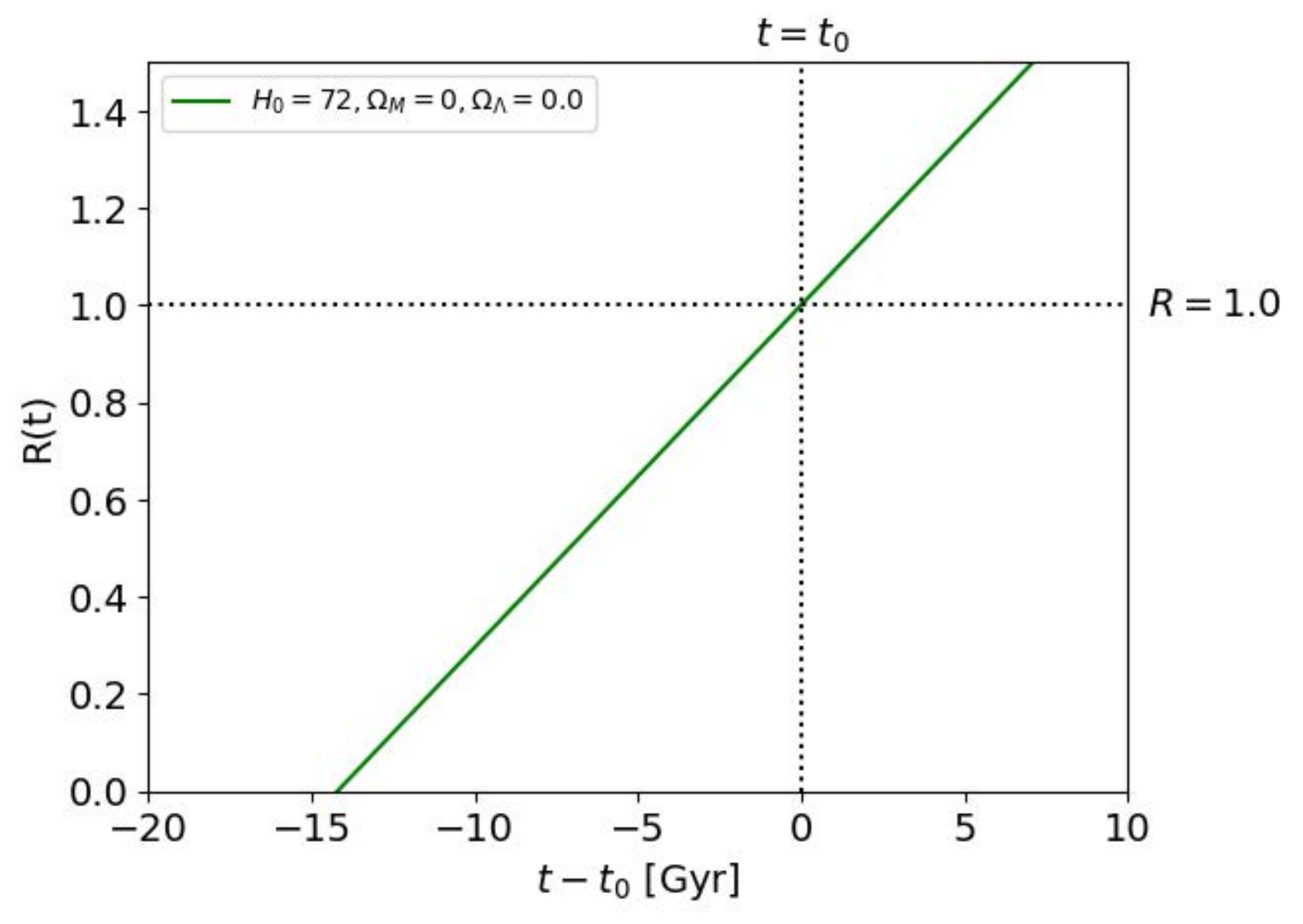

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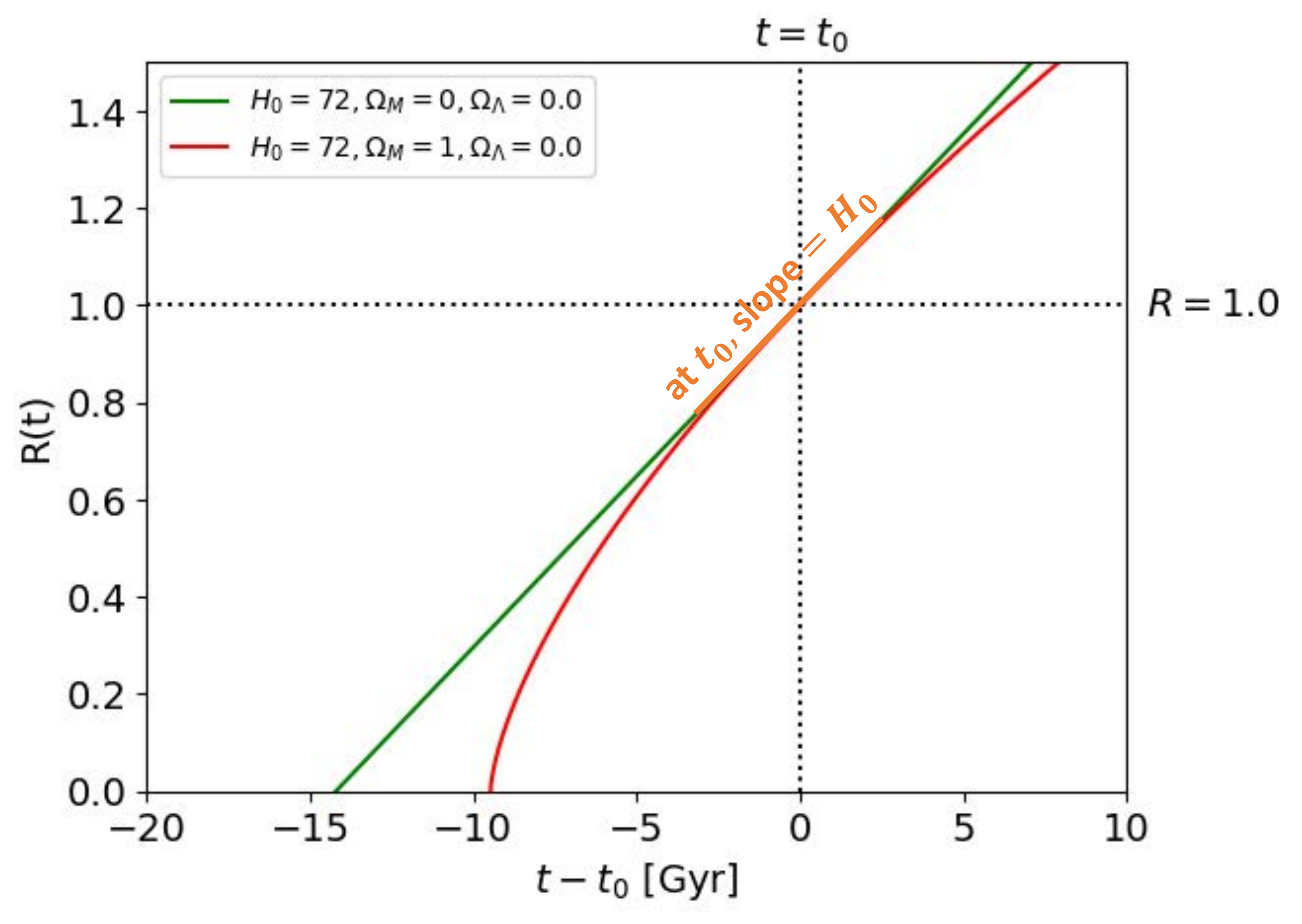
The more mass you put in (bigger Ω_m) the more the curve bends (deceleration due to gravity is getting strong), and the younger the Universe gets.

For $\Omega_m = 1, \Omega_\Lambda = 0, \dot{R}(\mathrm{t}) \longrightarrow 0$ as $\mathrm{t} \longrightarrow \infty$



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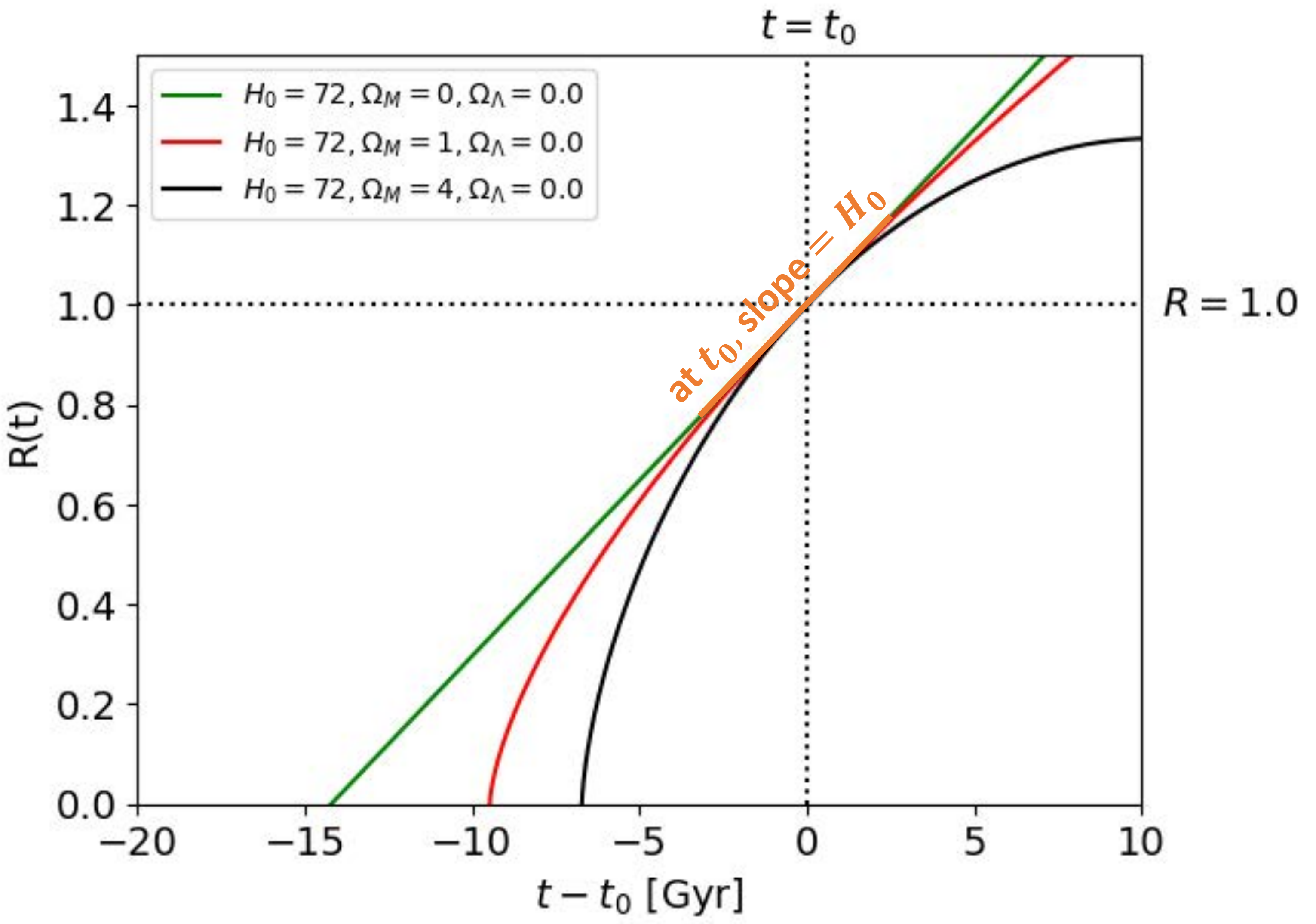
For $\Omega_m > 1, \Omega_\Lambda = 0$, the Universe eventually recollapses.



The R(t) plot: Understanding the parameters graphically and intuitively

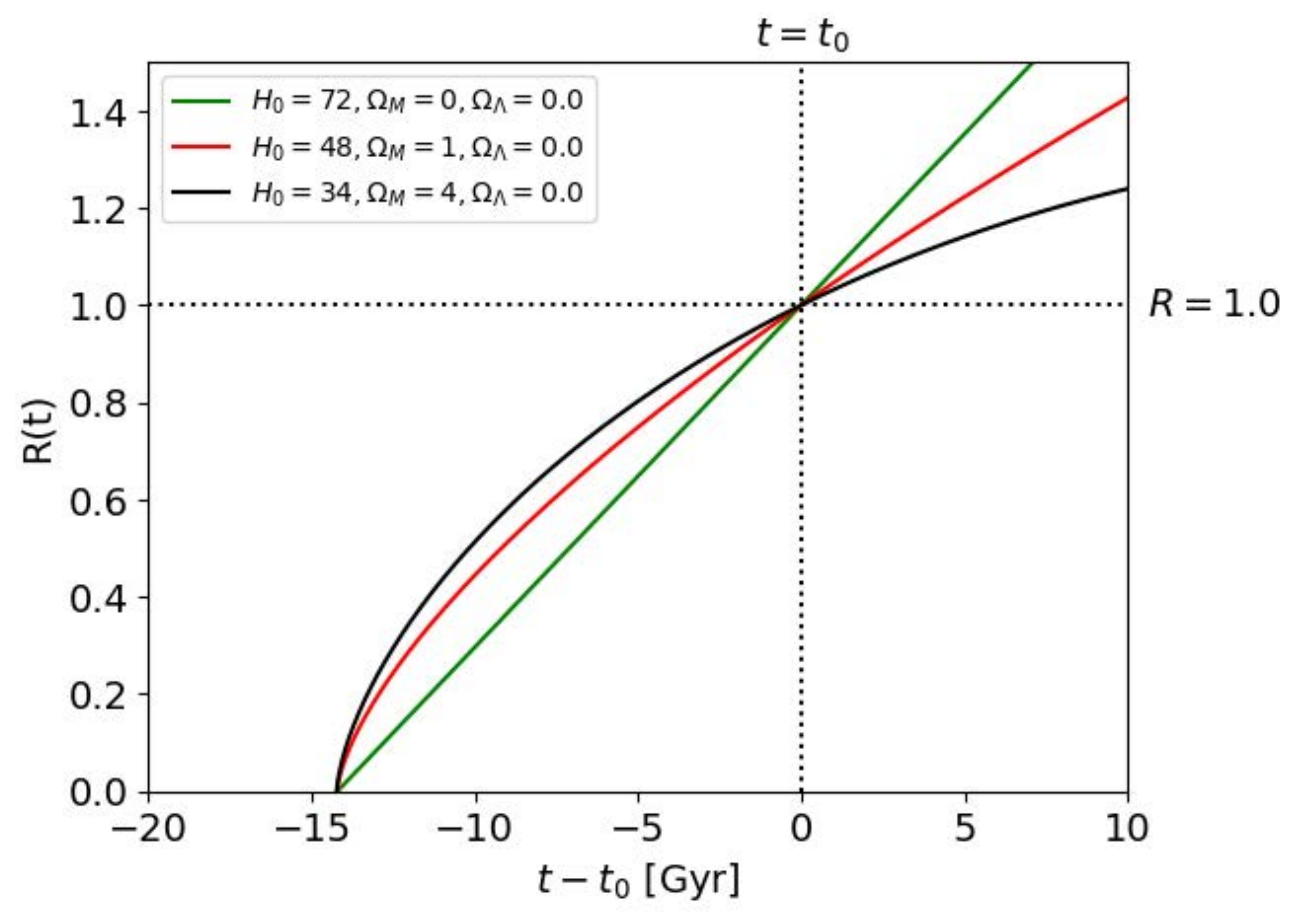
Now add matter to the Universe (so $\Omega_m > 0$) but no cosmological constant ($\Omega_\Lambda = 0$). Matter has gravity, and gravity slows down the expansion: deceleration. The Universe must have been expanding faster in the past, so R(t) must be "bending downwards."

What if we want the same age?

If I wanted to add mass but keep the age the same, I have to change the Hubble Constant H_0. So I'm changing the slope at $t = t_0$.

Three universes with different amounts of matter but having the same age due to different Hubble Constants. ⇒

But we have observational constraints on the Hubble Constant, so we are not free to do just anything we want with it!



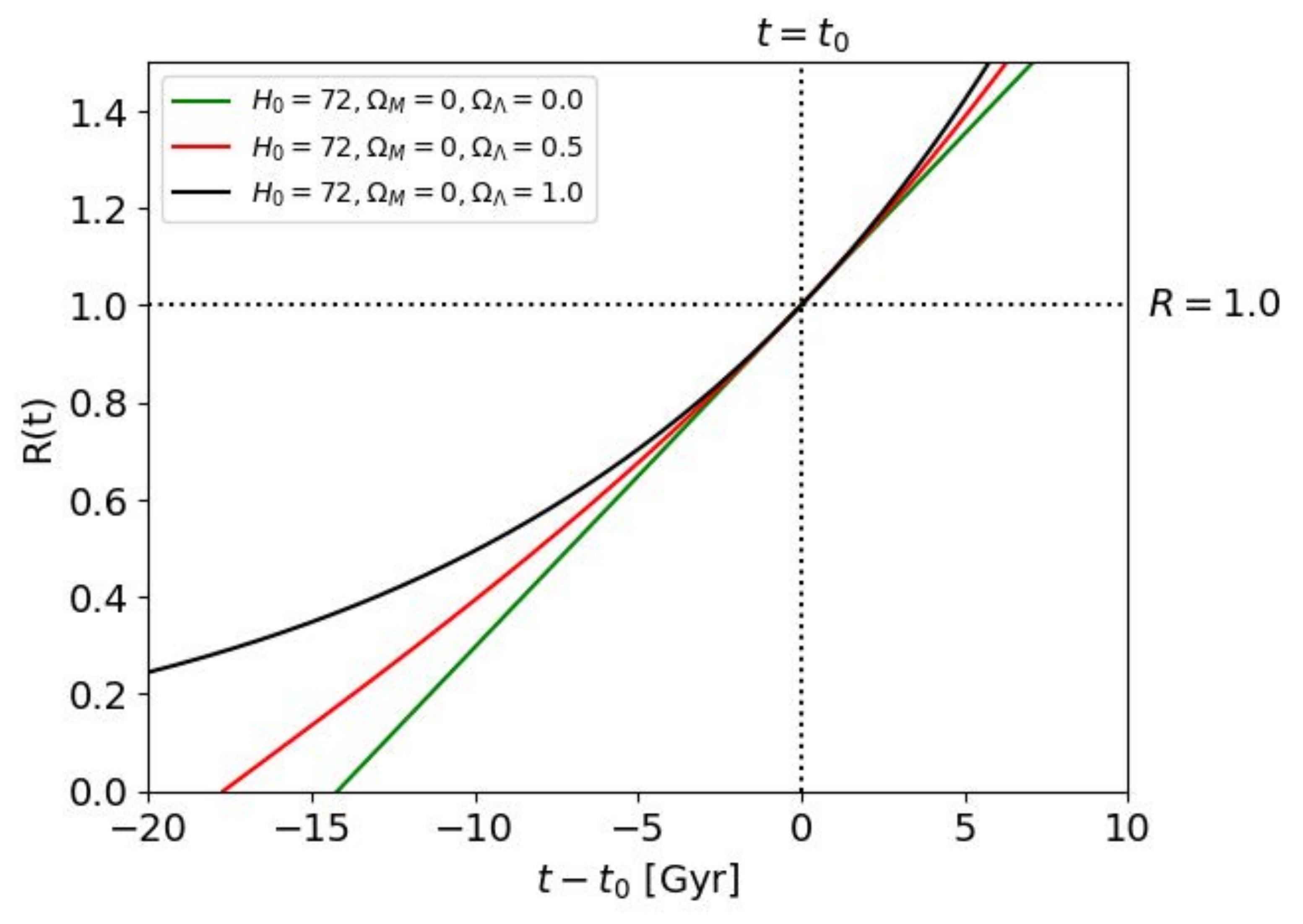
The R(t) plot: Understanding the parameters graphically and intuitively

Now remove matter and add a cosmological constant to the Universe (so $\Omega_m = 0,\ \Omega_\Lambda > 0$). This accelerates the expansion of Universe. It must have been expanding slower in the past, so R(t) must be “bending upwards.”

Again, since we have to obey the boundary conditions (R_0, H_0), an accelerating universe must be older than a constant expansion universe.

In fact, if we add too much lambda, we run into the problem of a universe that never has a beginning!

🤯



The R(t) plot: Understanding the parameters graphically and intuitively

What if we have both matter and a cosmological constant? So $\Omega_m > 0$ ***and*** $\Omega_\Lambda > 0$. Now we have a competition between matter decelerating the Universe and lambda accelerating the Universe. Who wins?

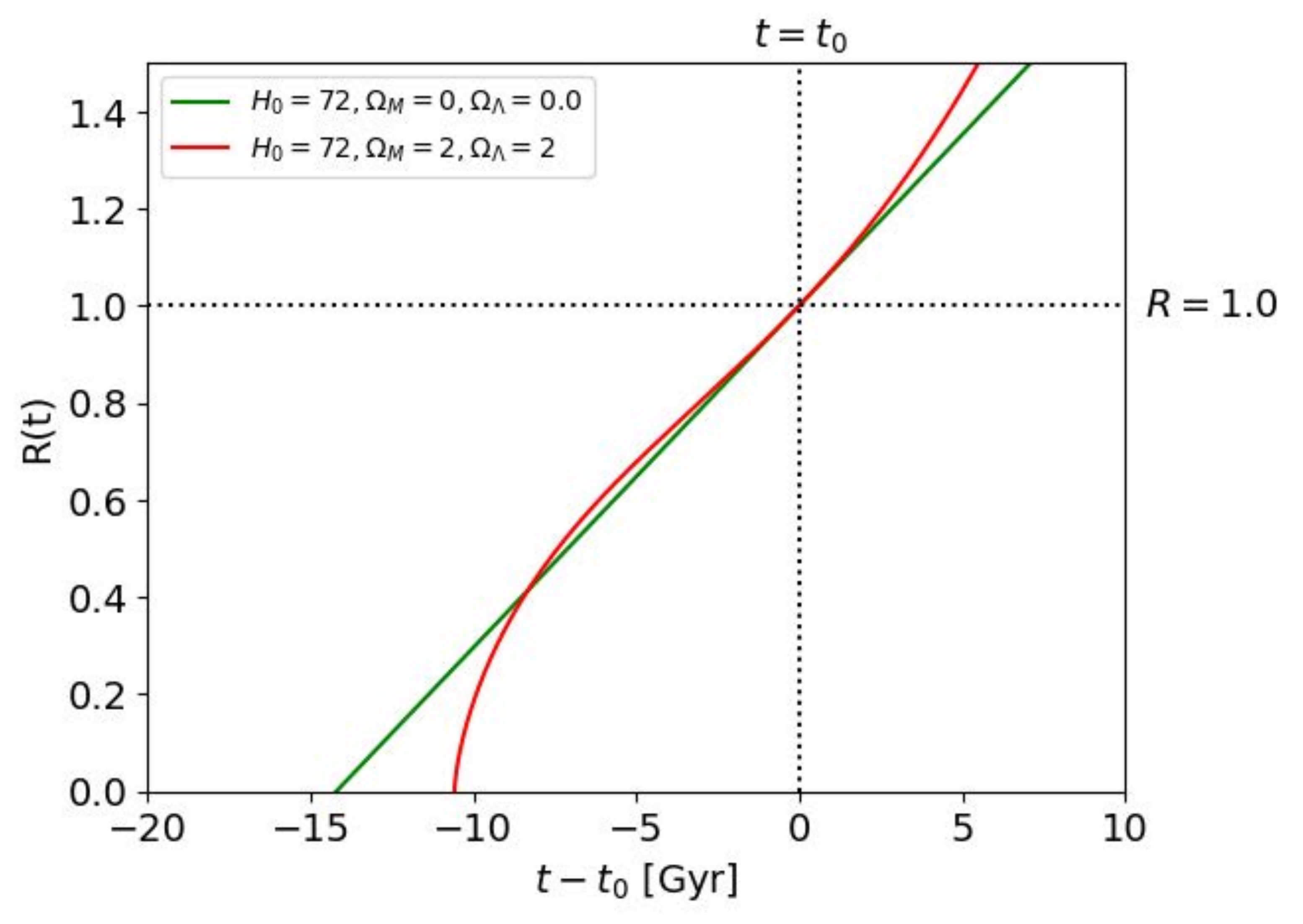
Go back to the Friedmann Equation:

$$\left(\frac{\dot{R}}{R}\right)^2 - \frac{8}{3}\pi G\rho - \frac{1}{3}\Lambda c^2 = -\frac{kc^2}{R^2}$$

Back in time, the universe was smaller, the density was higher, and gravity wins: the Universe starts out decelerating.

Over time, the density drops, gravity starts to lose, and lambda starts to dominate: late acceleration.

But remember, we still have to obey the boundary conditions (R_0, H_0)



Universes on the $(\Omega_{\text{m}}, \Omega_\Lambda)$ plane

These different behaviors can be mapped onto an $(\Omega_m, \Omega_\Lambda)$ plane to describe the resulting universes.

Remember the governing equations of cosmology. The **Dynamics Equation**:

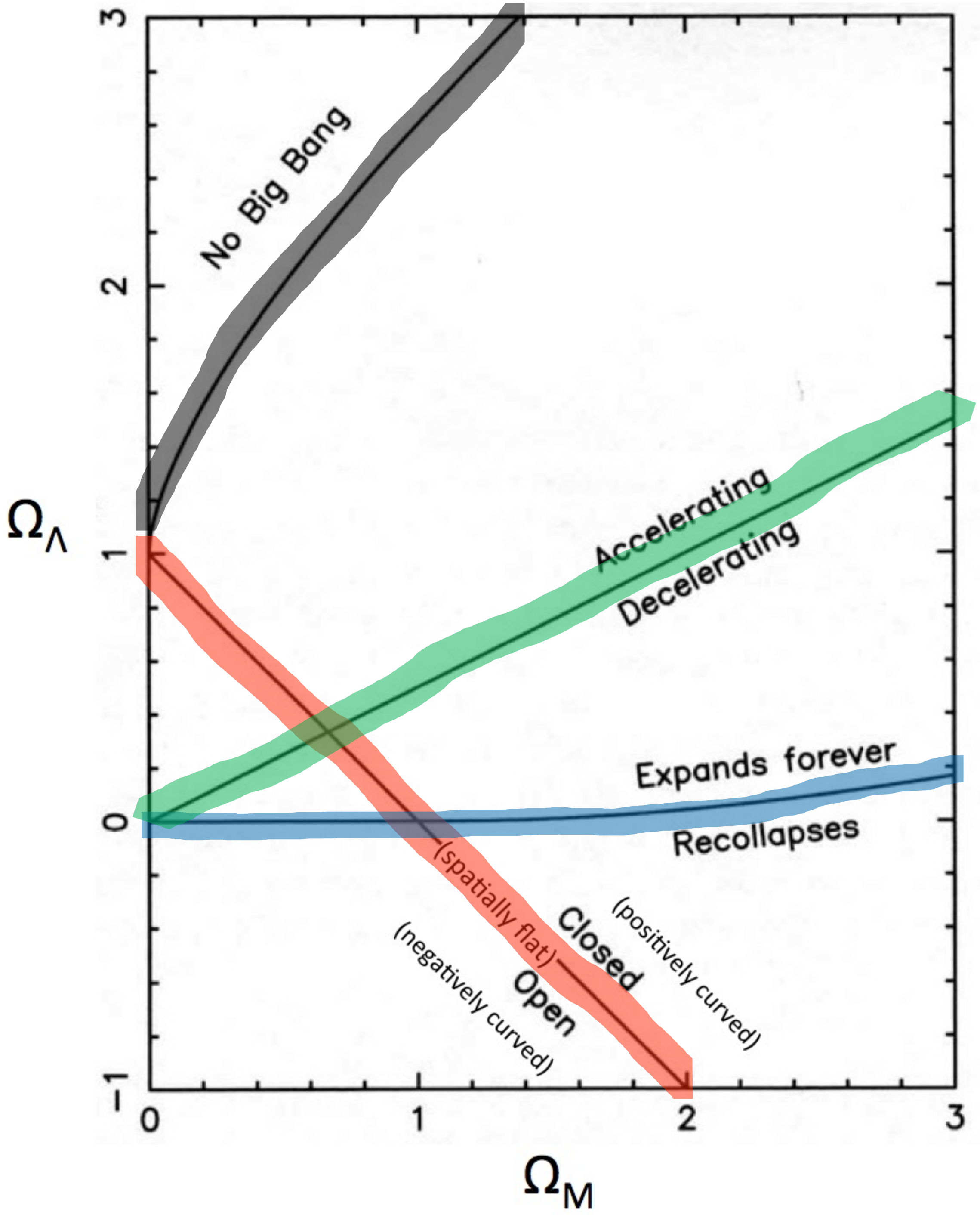
$$\frac{\ddot{R}}{R} = -\frac{4\pi}{3}G\rho + \frac{1}{3}\Lambda c^2$$

The Dynamics Equation shows that Λ (acceleration) works in the **opposite** sense of ρ (deceleration) in determining the expansion history.

The Dynamics Equation solves to the **Friedmann Equation:**

$$\left(\frac{\dot{R}}{R}\right)^2 - \frac{8}{3}\pi G\rho - \frac{1}{3}\Lambda c^2 = -\frac{kc^2}{R^2}$$

The Friedmann Equation shows that Λ and ρ work **together** in determining the curvature.

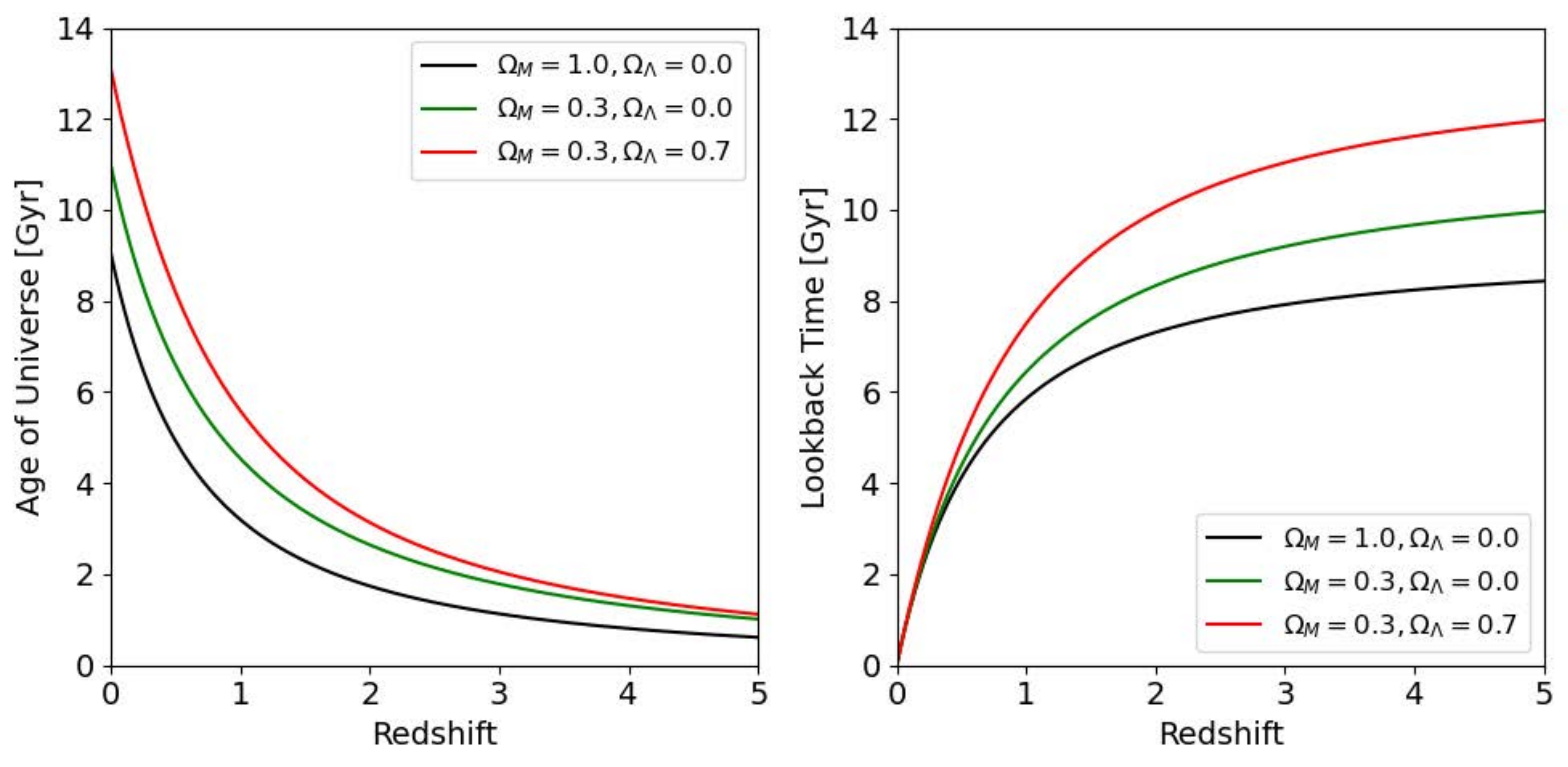


Ages and Lookback times

In all universes, redshift tells you the relative size of the Universe at that redshift: $\mathrm{R} = 1/(1+z)$.
But the connection between redshift and time is different in different cosmologies.

- Age: How old the universe was at a given redshift: $t(z)$
- Lookback time: How far back in the past are we looking at a given redshift: $t_0 - t(z)$

For $H_0 = 72$ km/s/Mpc ⇒



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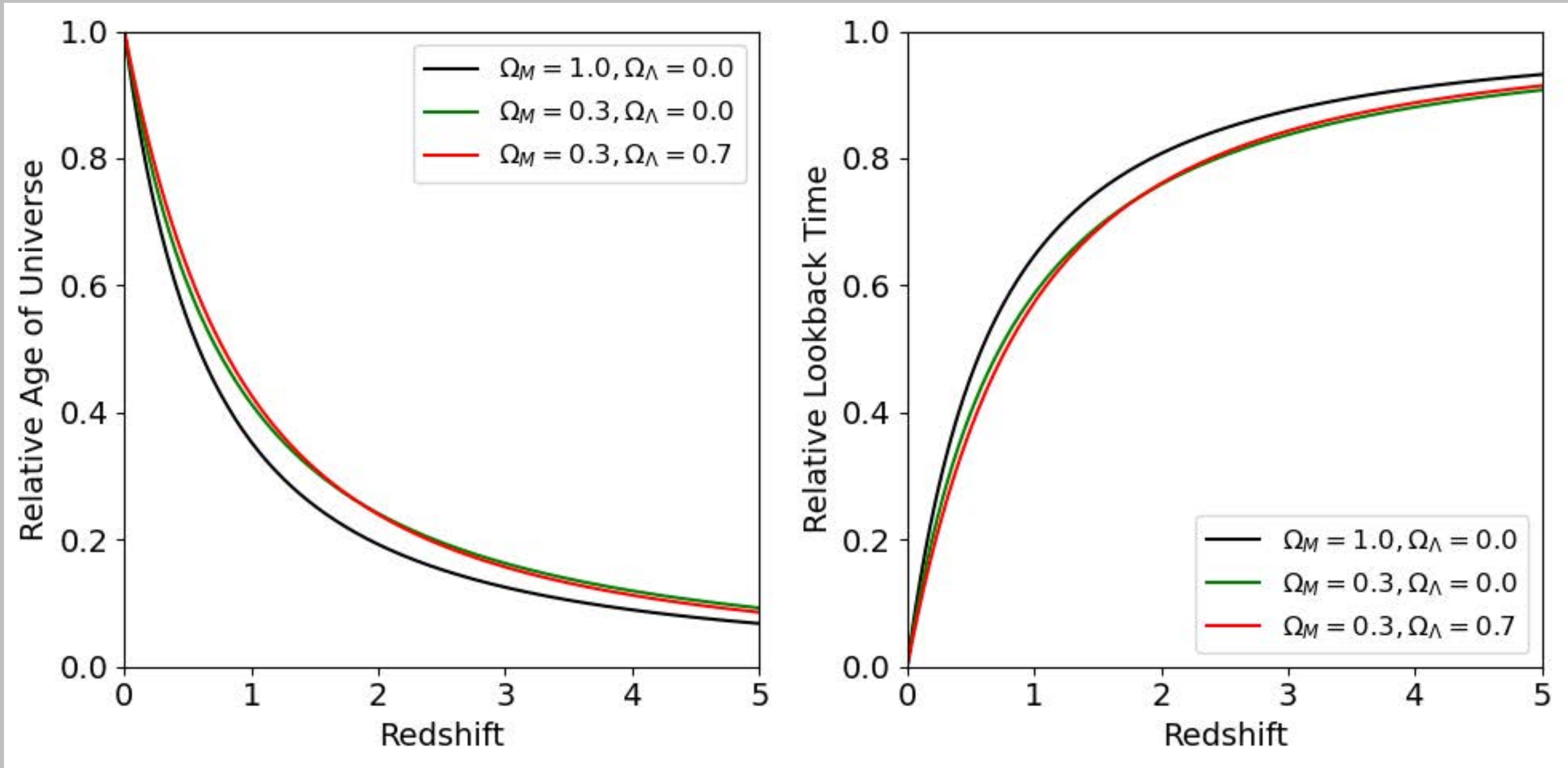
- Age: How old the universe was at a given redshift: $t(z)$
- Lookback time: How far back in the past are we looking at a given redshift: $t_0 - t(z)$

Here it is in relative terms, i.e., fraction of the Universe's current age.

In general:
$z = 1$: ≈ halfway back
$z = 3$: ≈ 85% back

(JWST seeing things at $z > 10$: ≈ 95% back!!!)



The Big Bang


Fred Hoyle

The Friedmann Eqn shows that the universe must be expanding from a very dense and hot initial state.

(Fred Hoyle was skeptical of this notion and in 1949 referred to it derisively in a BBC radio interview as “The Big Bang”. The name stuck.)

Remember, though, the Big Bang is not an explosion of material ***into*** space, its an expansion ***of*** space, carrying material with it.

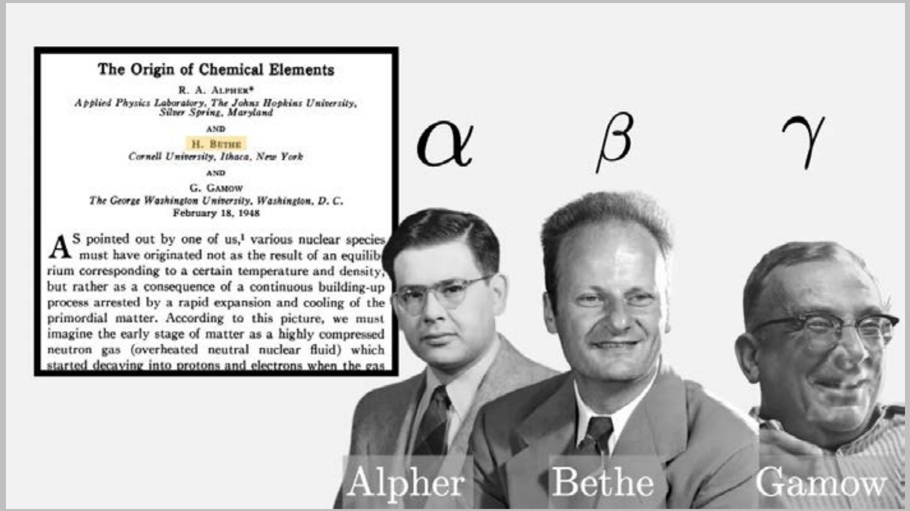
A dense hot object emits blackbody radiation, which peaks at a temperature given by Wien’s law:

$$\lambda_{peak} = \frac{0.29\ cm}{T\ (K)}$$

As the Universe expands, this blackbody spectrum is redshifted, but keeps the blackbody shape with a temperature that scales inversely with size: $T \sim 1/R$.

We say the Universe cools as it expands, and we should see this redshifted light from the Big Bang coming from all directions.

The Big Bang, nucleosynthesis, and the microwave background



1948: George Gamow and his student Ralph Alpher show that in the early universe the temperature and density would be right to fuse hydrogen into helium, at about the right He:H abundance ratio. They called this **"Big Bang Nucleosynthesis"** and added Hans Bethe to the study, writing the famous "$\alpha\beta\gamma$ paper".

Necessary conditions: $T_{BBN} \approx 10^9 K, \rho_{BBN} \approx 10^{-5}$ gm/cm^3.

Since density scales inversely with volume ($\rho \sim R^{-3}$), given the current density of the Universe (ρ_0), they worked out that this would happen when the Universe was at a scale factor of

$$R_{BBN} \approx (\rho_0/\rho_{BBN})^{1/3} \approx 3\times10^{-9}$$

And if $T \sim 1/R$, the Universe today should have a temperature of about

$$T_0 \approx T_{BBN}(R_{BBN}/R_0) \approx 3 \text{ K}$$

A 3K blackbody peaks at microwave wavelengths, so today's universe should be bathed in **the microwave background**.

The Big Bang, nucleosynthesis, and the microwave background

So in 1948, the microwave background was predicted but not yet observed.

Early 1960s: Princeton scientists were developing a new microwave/radio observatory to search for these cosmic microwaves, when suddenly....

...they get a call from AT&T Bell Laboratories.

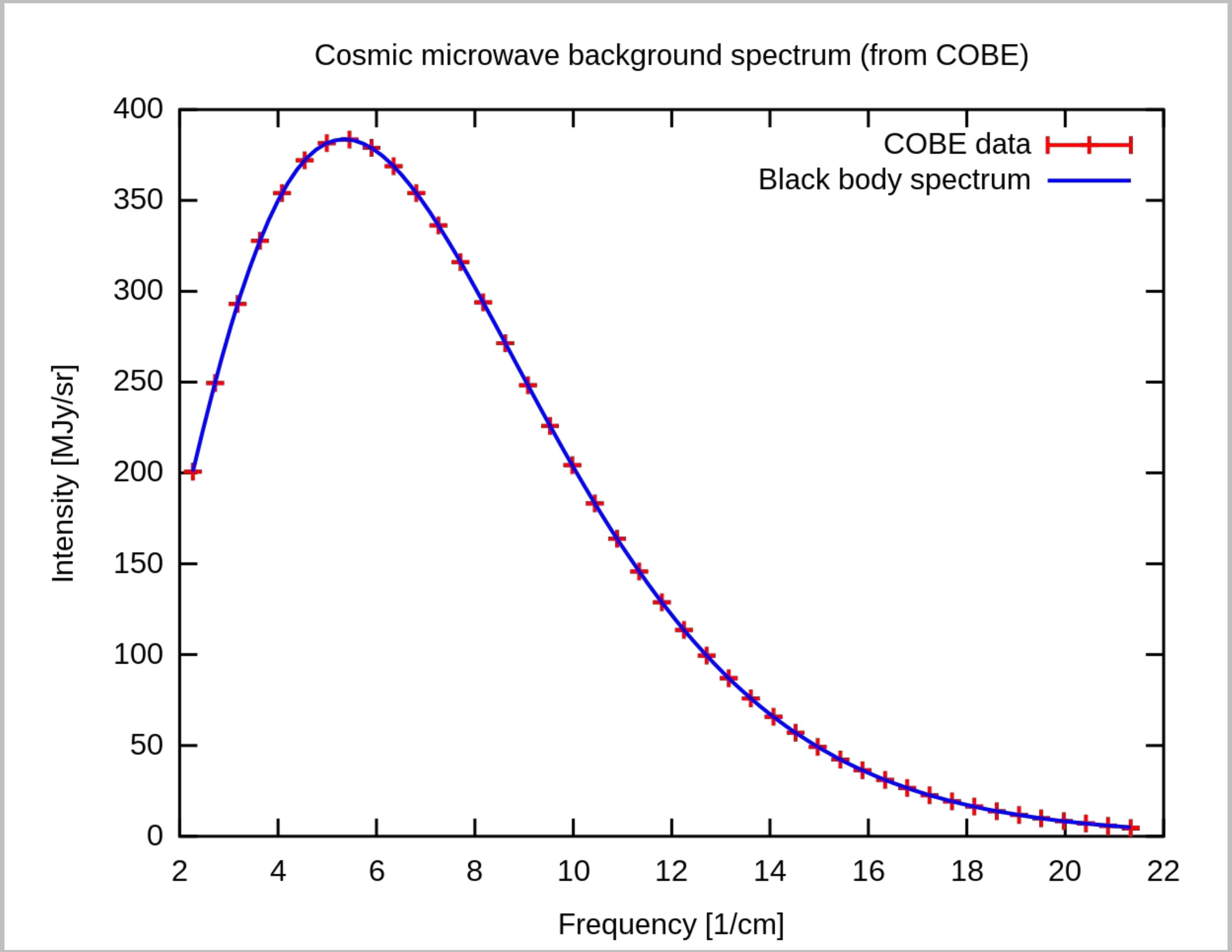
1964: Arno Penzias and Robert Wilson

Bell Lab engineers working on a radio antenna to communicate with the new Telstar satellites.

Report a persistant all-sky “hiss” in their equipment: the discovery of the cosmic micowave background (CMB).

The cosmic microwave background (CMB)

The CMB is a perfect blackbody with a temperature of 2.726 K.

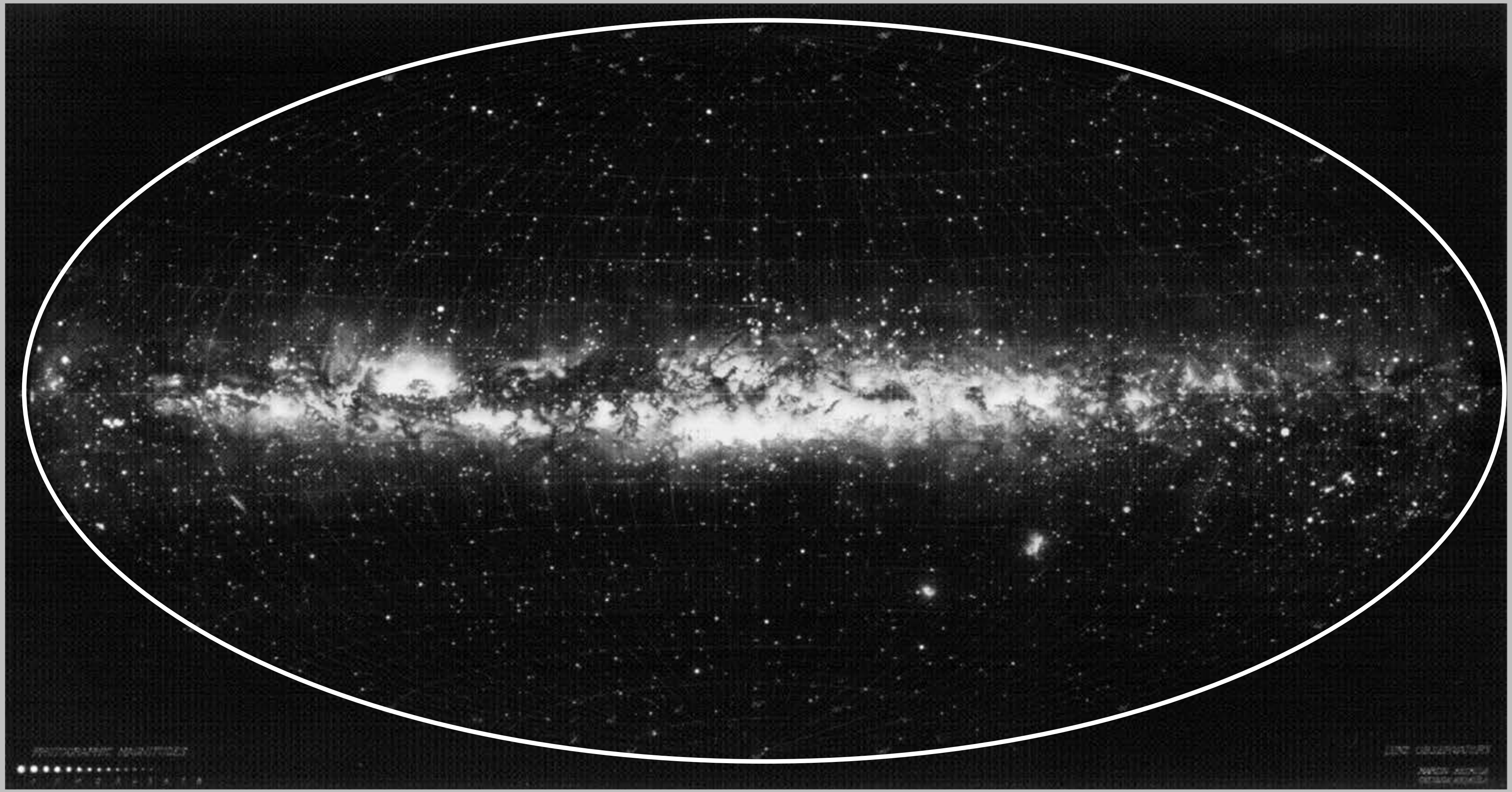


COsmic Background Explorer (COBE)
1989-1993

The cosmic microwave background (CMB)

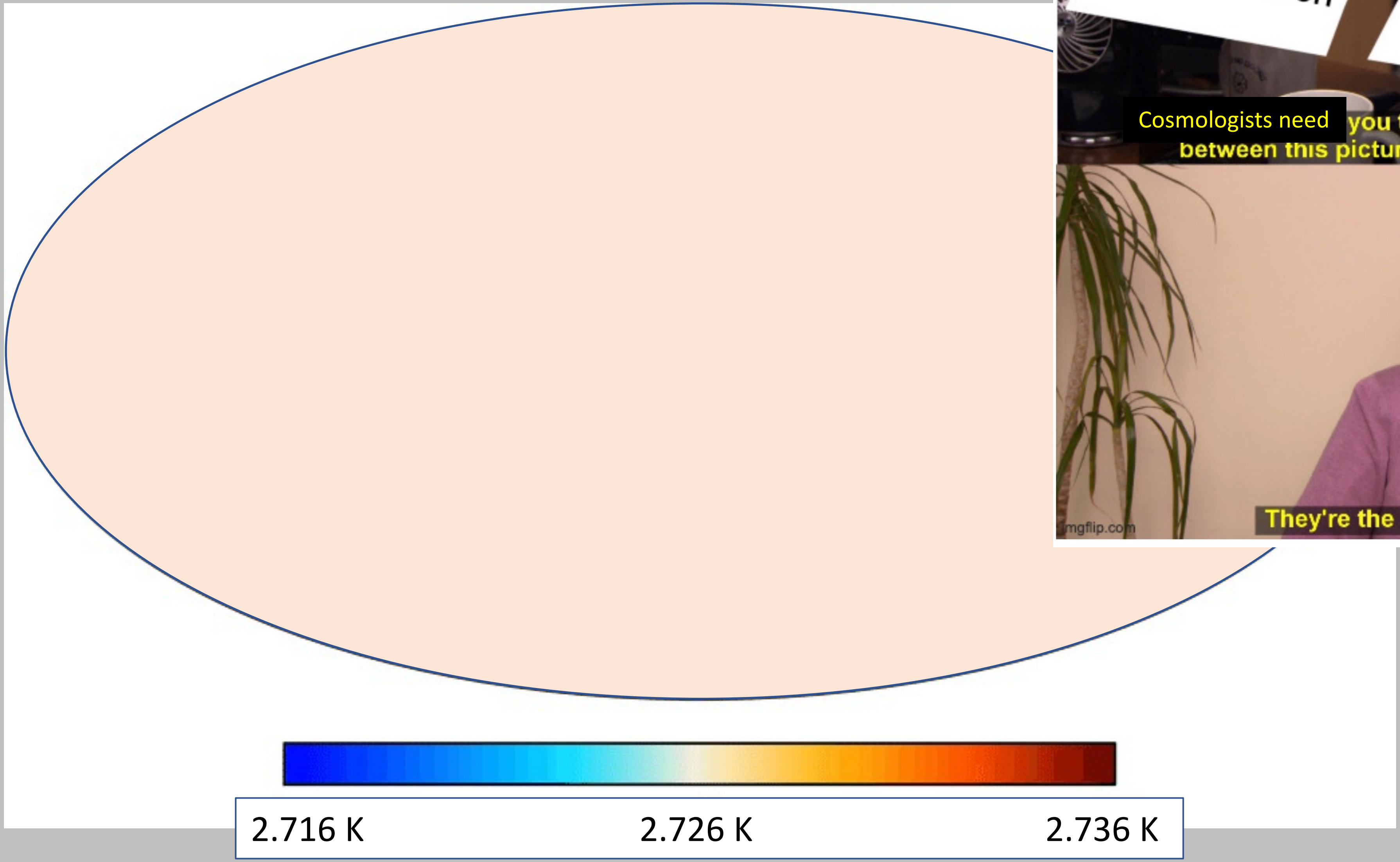
The CMB is a perfect blackbody with a temperature of 2.726 K.
The CMB is (almost) perfectly smooth.

all-sky optical map



The cosmic microwave background (CMB)

The CMB is a perfect blackbody with a temperature of 2.726 K.
The CMB is perfectly smooth (± 0.01 K).



The cosmic microwave background (CMB)

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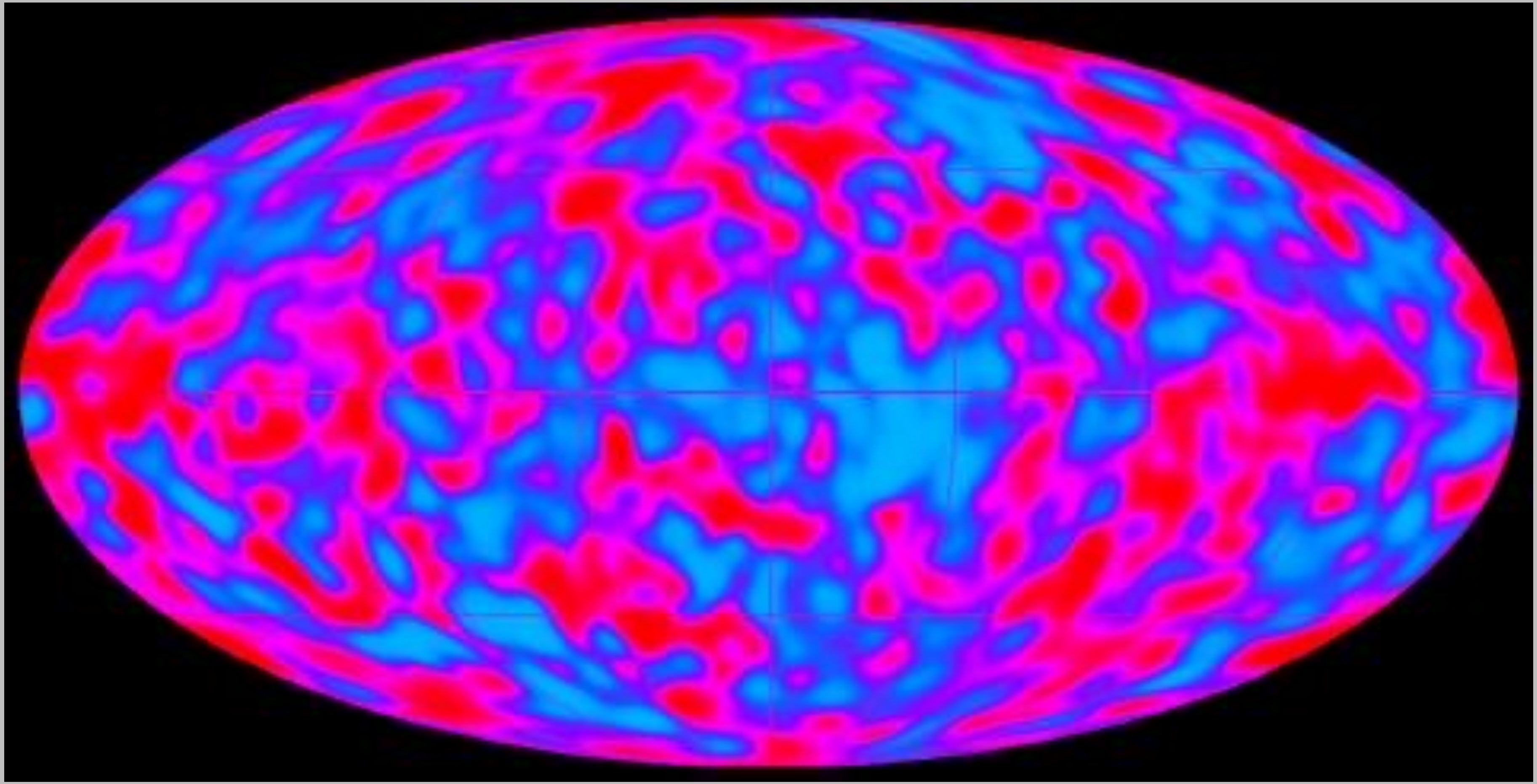
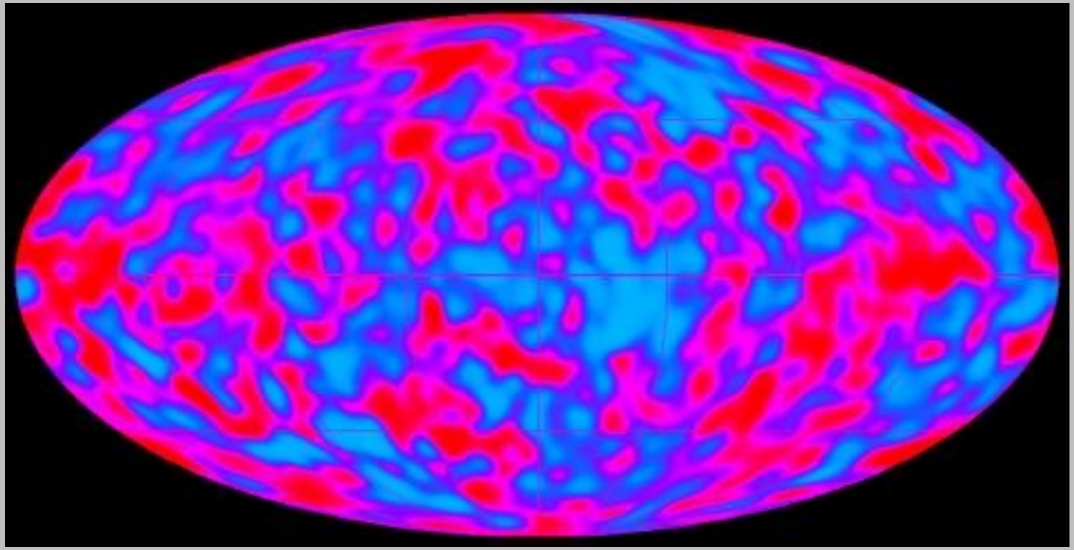
The CMB is perfectly smooth (± 0.01 K).
Well, almost perfectly smooth (± 0.0003 K)

COBE all-sky microwave map (1992)



The cosmic microwave background (CMB)

What are we actually seeing?



This is light from the very early Universe, redshifted into microwave frequencies.

In the hot, dense early Universe, everything was ionized, lots of free protons and electrons. Photons couldn't travel very far before being scattered by the electrons, so the Universe was **opaque**.

But the Universe is expanding and cooling, and at some point the temperature drops low enough that electrons and protons can bind together to form bound atoms ("**re-combination**"). At this point there are no more free electrons, and photons can travel freely: the Universe becomes **transparent**.

This happens when the temperature drops to ≈ 3000 K, which corresponds to a redshift of z ≈ 1000, or an age of ≈ 350,000 years after the Big Bang. We cannot "look back" to earlier times, because the Universe was opaque earlier than this.

The small temperature fluctuations correspond to regions of higher or lower mass density: the "lumps and bumps" of mass that will grow up to be galaxies and galaxy clusters: the large scale structure we see today.