ASTR 222 Homework #3

Population Synthesis (20 points)

We are going to make galaxies by mixing together different types of stars, shown in the table below. For each type of star, describe in words what kind of star it is: what evolutionary stage it is in, what kind of lifetime it has, what limits (if any) you can place on its age. Then calculate its stellar mass-to-light ratio $(\mathcal{M}/L)_*$.

	Star 1	Star 2	Star 3	Star 4
Spectral Type	A2V	G2V	K5V	K2III
Absolute Mag (M_V)	1.3	4.8	7.35	0.5
$B - V$ color	0.05	0.65	1.15	1.16
Mass ($\mathcal{M}_\odot$)	2	1	0.67	1.1
Mass-to-Light Ratio $(\mathcal{M}/L)_*$	*(fill this line in for each star!)*			

(Remember that mass-to-light units are solar units, so the Sun has a mass-to-light ratio of $(\mathcal{M}/L)_* = 1\ \mathcal{M}_\odot/L_\odot$. And also remember that the Sun has a V-band absolute magnitude of $M_V = +4.8$ and a color of $B - V = 0.65$.)

Now let's build some galaxies. The galaxies should each have a total V-band luminosity of $L_V = 10^{10}\ L_\odot$. The fraction of the total V-band light each star contributes to each galaxy is given in the table below. Calculate the total ("integrated") $B - V$ color and V-band stellar mass-to-light ratio $(\mathcal{M}/L)_*$, as well as the fraction of each star by number for each model galaxy. Show your work, by explaining a calculation for Galaxy 1 in exquisite detail.

	Fraction of V-band light from each star			
	Star 1	Star 2	Star 3	Star 4
Galaxy 1	15%	40%	25%	20%
Galaxy 2	30%	0%	0%	70%
Galaxy 3	45%	25%	20%	10%
Galaxy 4	0%	30%	70%	0%
Galaxy 5	0%	30%	50%	20%

Now, a "typical" color for a spiral galaxy like the Milky Way is $B - V = 0.7$, an elliptical might have a color of $B - V = 1.0$, and a starburst galaxy (that is forming stars at a furious rate) might have $B - V = 0.4$.

Which of these galaxies is a good match for an elliptical, which for a spiral, and which for a starburst? Which two galaxies don't make sense? Argue your answer both from integrated colors and from the mix of stellar types.

Surface brightness (10 points)

If a galaxy is observed face on (with no dust) and has a surface brightness equivalent to one solar-type star per square parsec, show that the surface brightness in magnitudes per square arcseconds is $\mu = 26.37$ mag/arcsec^2 in the V band.

From this, we can express the relationship between surface brightness and luminosity density as $\mu = 26.37 - 2.5\log(I)$, where I is the luminosity density of the galaxy in solar luminosities per square parsec.

If a galaxy has a central surface brightness of $\mu_V = 21.0$ mag/arcsec^2, what is the luminosity density in the center of the galaxy? We are now discovering really faint "ultradiffuse galaxies" that have central surface brightnesses of $\mu_V = 27.0$ mag/arcsec^2 -- what does that correspond to in terms of luminosity density?

Disk Galaxies: Luminosity, Rotation Curves and Dark Matter (20 points)

If a disk galaxy has an exponential surface brightness profile $I = I_0 e^{-(R/h)}$, where h is the scale length and I_0 is the central luminosity density (in $L_\odot$/pc^2), calculate:

1. the total luminosity of the galaxy (in terms of I_0 and h).
2. the half-light radius of the galaxy (in terms of h)
3. the radius containing 98% of the total light (in terms of h).

If the galaxy has a constant stellar mass-to-light ratio $(\mathcal{M}/L)_*$, derive an ***analytic expression*** for what the rotation curve of the galaxy should look like (ignore the bulge and halo of the Galaxy for this calculation). For your expression for the rotation curve, I want something that looks like $V_c(R) = f(I_0,\ (\mathcal{M}/L)_*, h, R)$.

Okay, now if the galaxy has a central surface brightness $\mu_0 = 19.2$ mag/arcsec^2, $(\mathcal{M}/L)_* = 1.0\ \ \mathcal{M}_\odot/L_\odot$, and $h = 3.5$ kpc, plot what the rotation curve of the galaxy should look like over the range $R = 0$ to 30 kpc. *(Hint: if it looks flat, you screwed up.)*

Now, the observed rotation curve is roughly flat; at $R = 30$ kpc, the circular velocity is still $V_c \approx 220$ km/s. What is the mass needed to give this circular velocity? How much disk mass is there inside $R = 30$ kpc? So how much dark matter do we need? So inside $R = 30$ kpc, what percentage of the Galaxy's mass is in dark matter?

Some studies suggest that the rotation curve of the Galaxy remains flat out to 150 kpc (or further!). If so, what fraction of the Galaxy's mass (inside 150 kpc) is in the form of dark matter?

Exploring HII Regions (15 points)

Luminous HII regions around O and B stars are a very conspicuous part of the interstellar medium. These stars are the hottest, most luminous, and shortest-lived stars on the main sequence and thus spend their entire lives embedded in the high-density gas from which they formed. The large amount of ultraviolet radiation emitted by these hot stars ionizes the gas surrounding them, creating a bright HII region. In this problem, we're going to explore some of the physical properties of these HII regions.

Let's assume that the gas around a hot star consists of pure hydrogen. This means that the gas will be optically thick at wavelengths $\lambda < 912$ Å, corresponding to photons with energy $h\nu > 13.6$ eV, high enough to ionize hydrogen. In other words, photons with $\lambda < 912$ Å will ionize a single atom of hydrogen.

Now let's think physically about what is going on. We just ionized a hydrogen atom, creating a proton and a free electron. If that free electron immediately recombines with a proton, then nothing really happened. However, the density inside HII regions is very low and so there is some timescale (called the recombination timescale) involved when the free electron recombines with a proton. As soon as the hot star has ionized some hydrogen, its opacity to ionizing photons becomes much larger, so the photons can travel further distances and ionize more of the gas cloud.

Thus, the radius of the ionized sphere grows with time until an equilibrium between the ionization rate (which is a function of the temperature and luminosity of the ionizing star) and the recombination rate is reached. At that time, for every newly ionized hydrogen atom there is a newly creating hydrogen atom from recombination. Inside this spherical volume (called a Strömgren sphere, electrons and protons recombine at a rate of

$$N_{rec} \approx \alpha_H n_e n_p$$

where N_{rec} is the number of recombinations per unit time per unit volume (and has units of cm^{-3} s^{-1}), $\alpha_H = 4.2 \times 10^{-13}$ cm^3 s^{-1} is the recombination coefficient measuring the crosssection for the interaction between a proton and electron, and n_e and n_p are the number density (in cm^{-3}) of electrons and protons, respectively.

If the typical density of electrons and protons inside an HII region is $n_e \approx n_p = 10^3$ cm^{-3}, what is typical recombination rate for an HII region?

We can express the recombination time, i.e., the time it takes for free electrons to recombine with protons, as

$$t_{rec} = \frac{n_e}{N_{rec}}$$

Assuming the same electron density as before, what is the recombination time for an HII region in years? How does this compare to the lifetime of an O or B star? Do you think this region will reach equilibrium?

Now, let's assume that we have reached the equilibrium state of a Strömgren sphere. If we know the ionization rate of a star, Q_*, in photons per second, we can write

$$Q_* = N_{rec}V$$

where V is the volume of the Strömgren sphere. Solve this equation for R_S, the radius of the Strömgren sphere. Your equation should be in terms of n_e, n_p, Q_*, and constants. Use your resulting equation to find the sizes of HII regions (in parsecs) powered by the following stars, assuming $n_e \approx n_p = 10^3$ cm^{-3}:

- An O3V star with $Q_* = 10^{49.63}$ s^{-1}
- An O7V star with $Q_* = 10^{48.63}$ s^{-1}
- An O9V star with $Q_* = 10^{47.90}$ s^{-1}

Finally, let's say that you have discovered two objects near the spiral galaxy M101. Based on previous work, you believe one of them is an HII region and another is a star-forming dwarf galaxy. You estimate the number of ionizing photons required to produce these two objects, called A and B, as $Q_A = 3.28 \times 10^{48}$ s^{-1} and $Q_B = 6.23 \times 10^{49}$ s^{-1} respectively. How many O9V stars would be required to power these two objects? Which one is likely to be an HII region and which is likely to be a dwarf galaxy? Why?