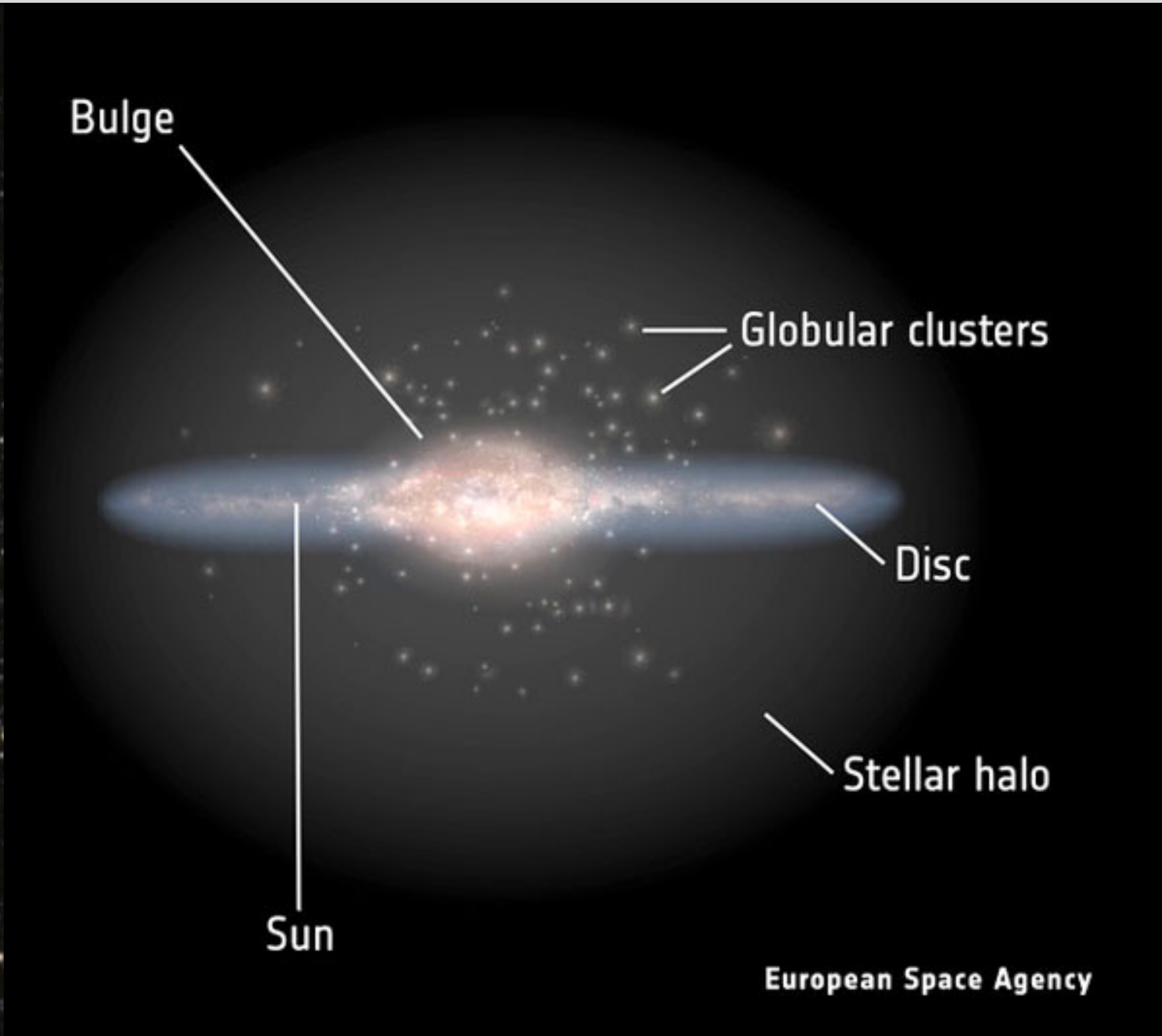
Problem #1: Studying the Milky Way Using Globular Clusters

Two types of globular clusters

- Disk globulars (near to the Galactic center, partly rotating with the disk, moderately metal-poor
- Halo globulars (spread throughout the halo, pure random motion, much more metal-poor



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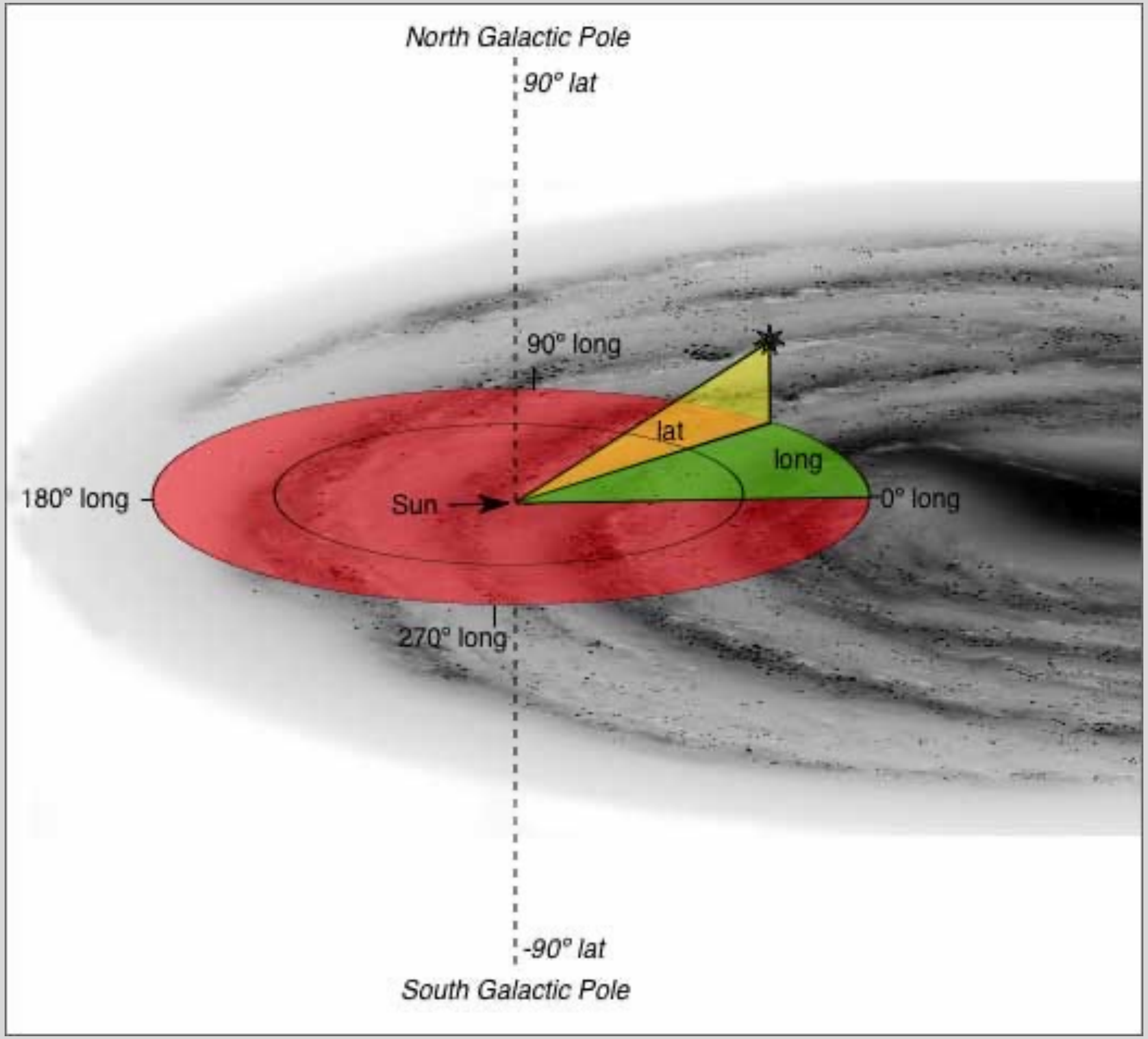
Data file gives:

- R: Distance, kpc
- feh: Metallicity, [Fe/H]
- Vr: radial velocity, in km/s
- Galactic coordinates:
  - l: Galactic longitude (l): Angle around in the disk plane
  - b: Galactic latitude: (b): Angle up/down out of the disk plane



Can convert Galactic coordinates to Cartesian coordinates like this:

$$X = R \cos l \cos b$$
$$Y = R \sin l \cos b$$
$$Z = R \sin b$$

Finding the Galactic Center

Shapley: the center of the globular cluster systems is the center of the Galaxy.

So if we can find the distance to the center of the globular cluster system, we know how far away the center of the Galaxy is.

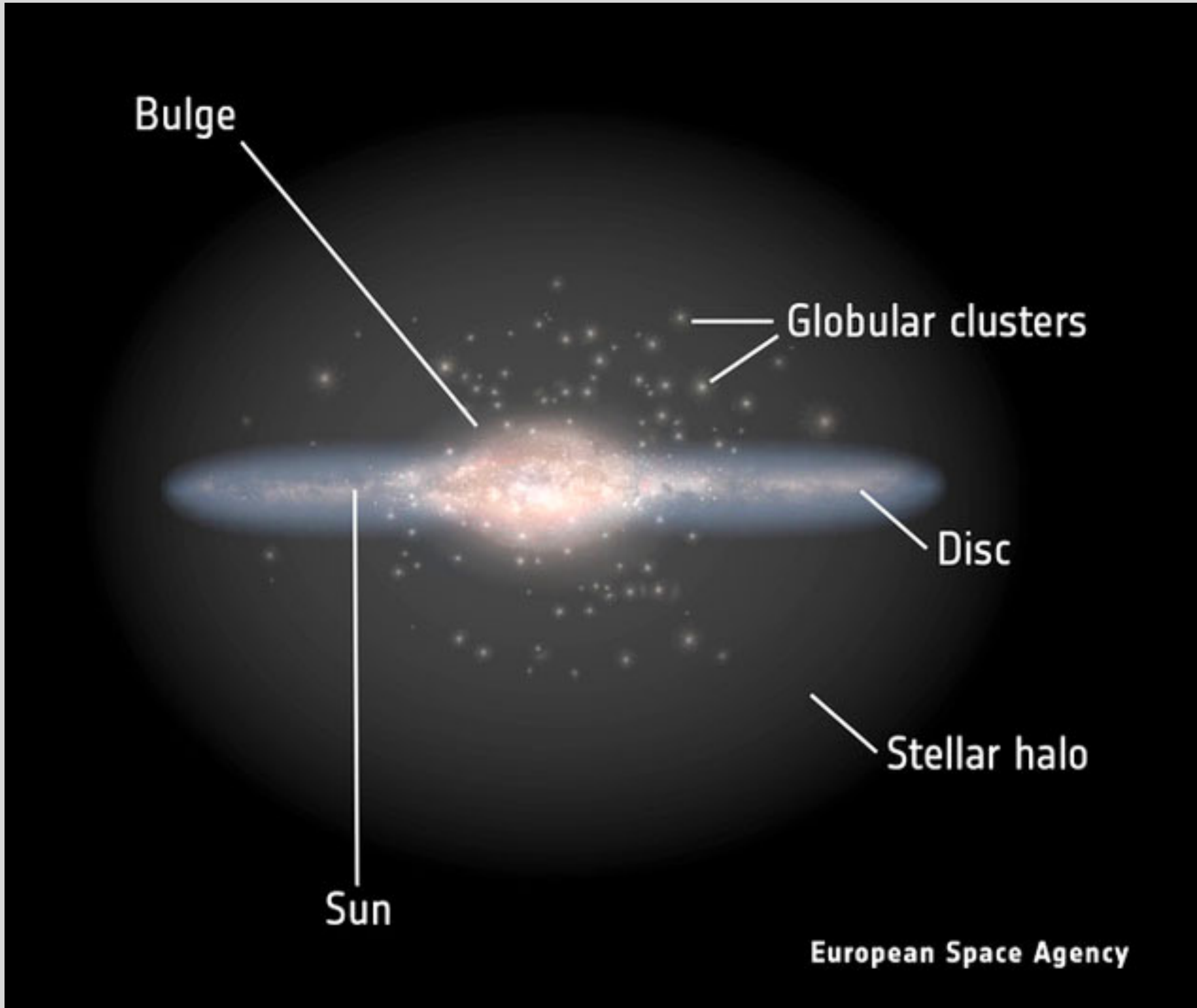
Steps:

- Plot XY and XZ projections for the metal-poor and metal-rich clusters – do the positions look right?

  Make sure your plots have the right aspect ratio: you are making a map!

- If the Sun is at (X,Y,Z)=(0,0,0), where is the center of the globular cluster system?

  Do the calculation separately for metal-poor and metal-rich clusters, and think about the differences.



What about the rotation speed of the disk? If the GCs are not rotating, we should be able to measure the speed of the Sun rotating around the Galactic center (V_c).

Imagine all the GCs are fixed in space (not moving) and in the disk plane. We should see a sine curve if we plot GC radial velocity vs GC galactic longitude.

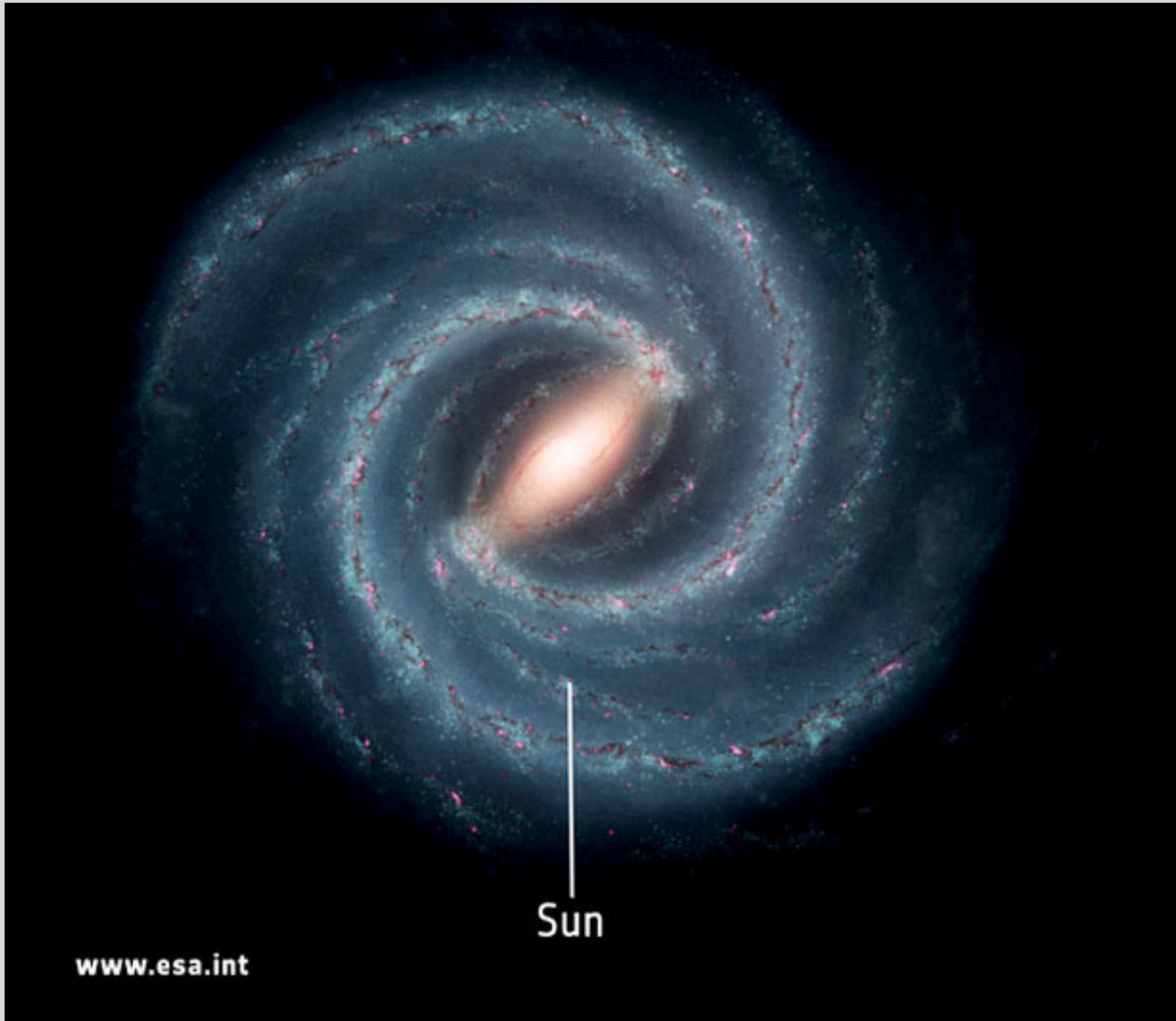
$$V_r = V_c \sin l$$

But because GCs are moving randomly, they will scatter around a perfect sine wave.

And also, GCs are not all in the disk plane, so that means the sine wave will be weaker.

Steps:

- Plot V_r vs l: Do we see a sine wave?
- Plot V_r vs $\sin l$: Fit a straight line, the slope will be a weakened measure of V_c
- Measure the scatter of the points around the sine wave, this will be a measure of the random velocity (velocity dispersion) of the GCs.



We need to correct these estimates of rotation velocity and GC velocity dispersion for projection effects: the fact that the observed radial velocity does not measure the full motion.

Correcting the rotation velocity

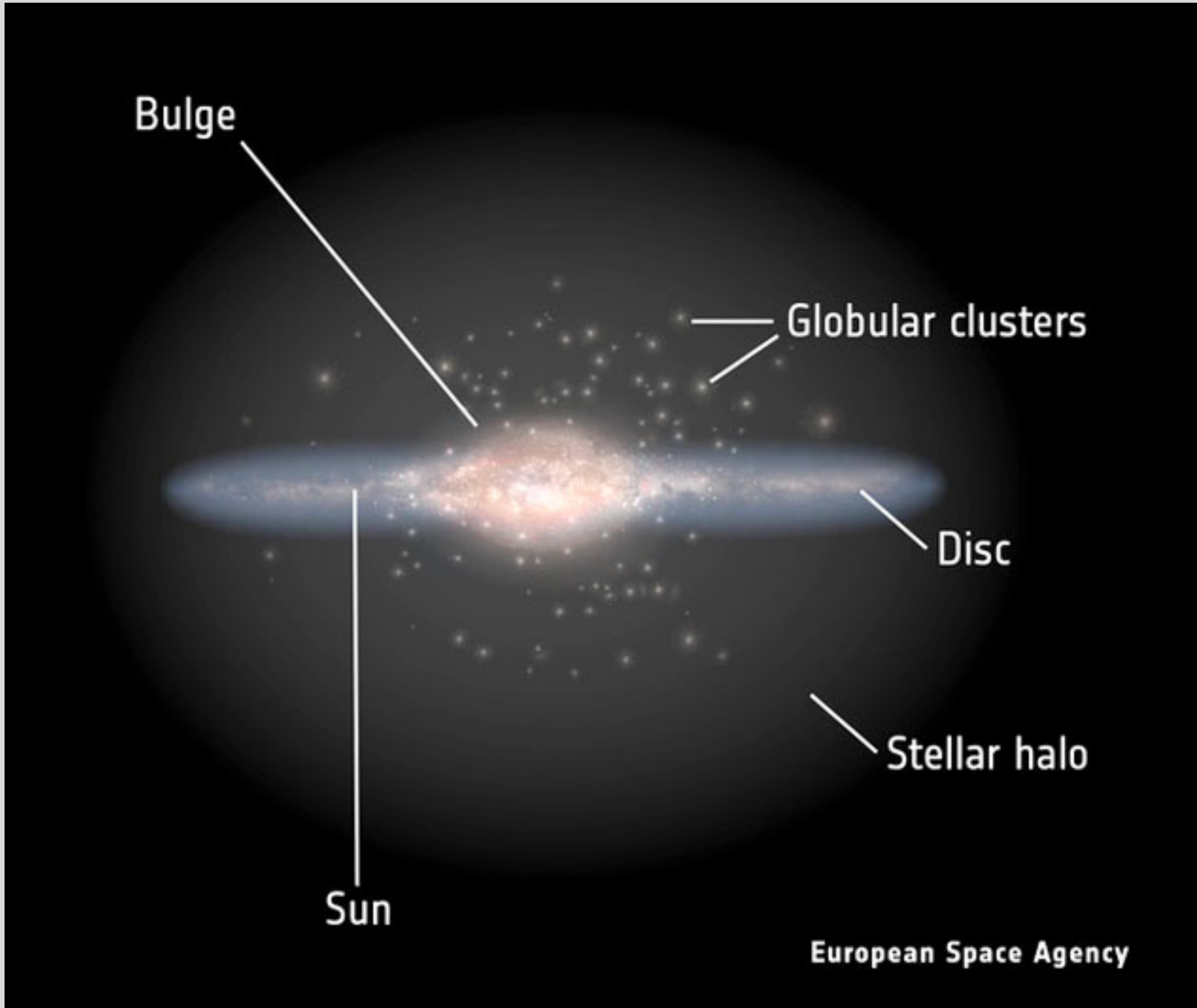
We measure a weakened circular velocity because we do not measure the true circular speed when we use GCs that are high above or below the Galactic plane.

If GCs are randomly distributed in space, the correction is just given by multiply your "weak" V_c by $\pi/2$ to get the true circular velocity.

Correcting the GC velocity dispersion

We measure the dispersion (scatter) of GC velocities only along our line of sight. This is called σ_r. But GC can move in all three directions. If the random motion is the same in all three directions, the velocity adds like this:

$$\sigma_{3D} = \sqrt{\sigma_r^2 + \sigma_\theta^2 + \sigma_\varphi^2} = \sqrt{3} \times \sigma_r$$



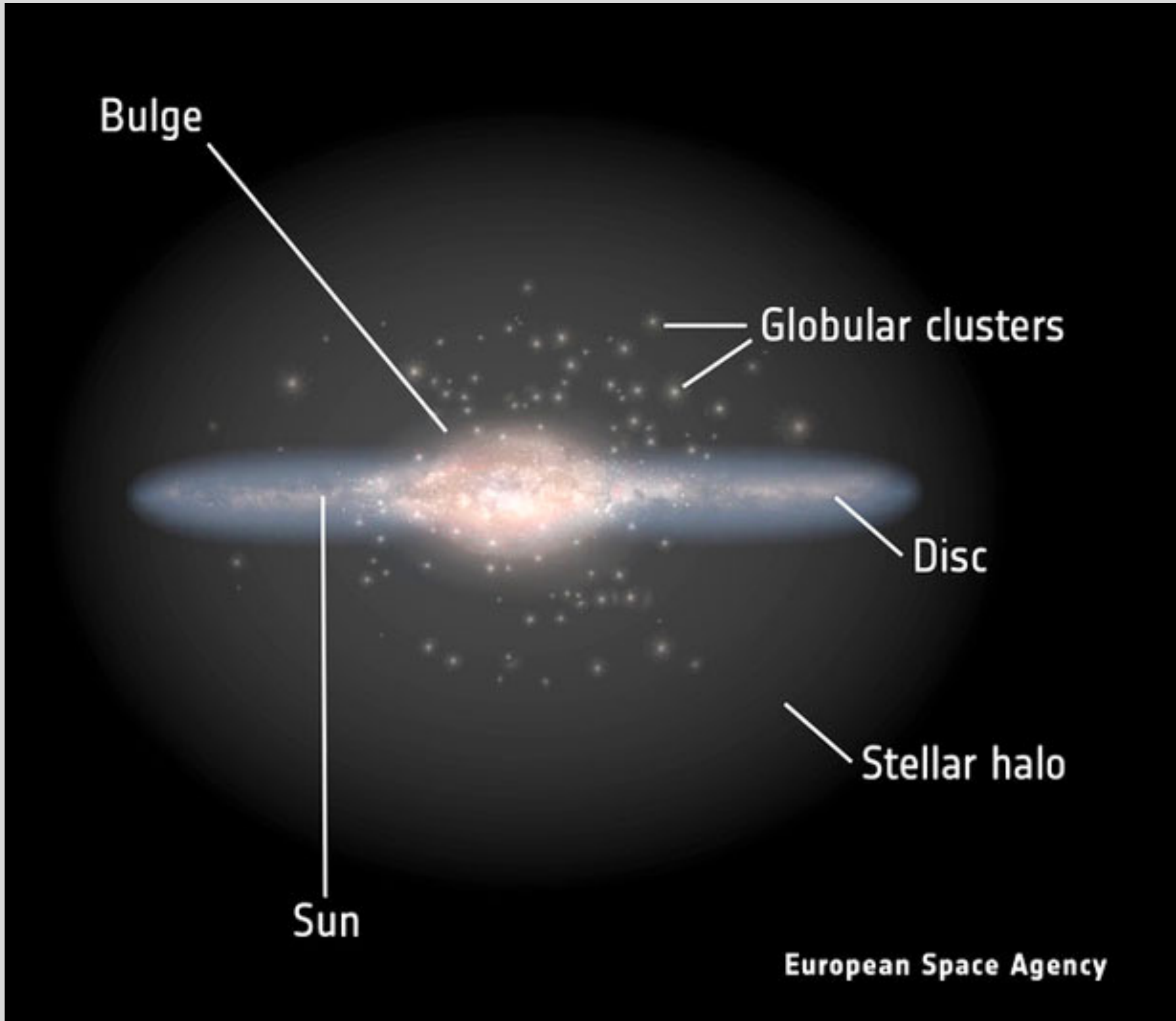
Problem #2: Using the Oort Limit to Calculate the Mass Density of the Galaxy's Disk

Remember the Oort Limit, which balances the vertical velocity dispersion of stars and the gravitational pull of the Galactic disk to work out the disk mass density.

$$\sigma_W^2 \sim 2\pi G \Sigma_0 h_z$$

So we need to measure the vertical velocity dispersion of stars (σ_W) as well as the thickness (scale height) of stars (h_z).

Then we can calculate the disk mass density, Σ_0



Star sample: Red giant stars observed at $b = -90°$ (directly down out of the disk plane from the Sun)

Measuring the thickness / scale height of the disk (z_0).

Remember that the vertical distribution of stars in the disk goes as e^{-z/z_0}. So if we plot the **natural log (ln)** of the number of stars as a function of distance, it should be a straight line with slope of $-1/z_0$.

Steps

- Get the distance to each star ($m - M = 5\log d - 5$). Since we are looking directly down, that distance ***is*** the z coordinate of the star (once we correct for the Sun's z coordinate)

- Count how many stars there are as a function of z. Make z bins of 200 pc in size, and count stars in each bin of z. Use scipy's binned_statistic to do this; look at thePython help "Binning/Fitting" page for help.

- Plot $\ln N$ vs z, it should be a straight line over some range of z. Fit the slope over the range on which it looks linear (not all the bins!) and derive the scale height, z_0, in parsecs.

Remember, in python np.log10() is base10 log and np.log() is natural log!

Measuring the vertical velocity dispersion of the stars (σ_W).

Since we are looking directly down, the radial velocity dispersion we see is the vertical velocity dispersion.

Steps

- From the observed velocities, calculate the Sun's W velocity and subtract that from all the velocities to get the stars' W velocity.

- Bin the stars in z again and calculate the W velocity dispersion (the standard deviation of velocities) as a function of z. Plot σ_W vs z and think about the plot. What is your best estimate for σ_W?

Calculate the mass density of the disk

Use the Oort Limit equation $\sigma_W^2 \sim 2\pi G \Sigma_0 h_z$ to calculate the mass density of the disk.

Remember, if h_z is in parsecs, and σ_W is in km/s, use the astronomical value of $G \approx 4.3\times10^{-3}\ \mathrm{pc\ (km/s)^2}\ M_\odot^{-1}$ and then your units of Σ_0 will be in $M_\odot/\mathrm{pc}^2$.