ASTR 222 (Spring 2023)
Homework #1

Star counts (10 points)

In star count lingo, N(m) is defined to be the number of stars observed with apparent magnitude brighter than m. As we saw in class, if the Galaxy had a uniform density of stars and is infinite in extent, we should see the star counts go as $\log N = 0.6m + C$ (this is true even if stars don't all have the same brightness).

Here is a table of integrated star counts $\log N(m)$ (again, $N(m) =$ the number of stars brighter than apparent magnitude m), taken from Allen's "Astrophysical Quantities." The columns are

1. mag: apparent magnitude (m)
2. logN_up: $\log N(m)$ in a direction up out of the galactic plane
3. logN_in: $\log N(m)$in a direction towards the galactic center

With this data, make a plot showing $\log N$ vs m in the two different directions. Make sure to plot both of them on the same plot! Also include on the plot the expected logN-vs-m relation for an infinite, uniform distribution of stars (given above; choose a value of C so that the line runs through the points for logN_up at bright magnitudes). Explain qualitatively why the star counts in different directions are different from each other, and also why they are different from the uniform model.

Note: when I say "plot this versus that", it means that "this" goes on the y-axis and "that" goes on the x-axis. So in the plot I'm asking you to make for this problem, log N goes on the y-axis and m goes on the x-axis.

Distance uncertainty (10 points)

Simple Gaussian error propagation says that if you measure a value x and use that to calculate a quantity y using the expression $y = f(x)$, then the uncertainty in x leads to an uncertainty in y that is given by

$$\sigma_y = \sigma_x \left(\frac{\partial f}{\partial x}\right)$$

where σ_x and σ_y refer to the uncertainties in x and y, respectively. (Note that this only works for small fractional uncertainties…)

Use this expression to show that if you are using distance modulus (m-M) to get the distance to an object, if you have a small magnitude uncertainty of σ_m, you get a ***fractional*** uncertainty in distance (i.e., σ_d/d) of approximately $0.5\sigma_m$. In other words, as an example, if your distance modulus error is 0.1 magnitudes, your distance uncertainty is about 5%.

In a recent paper, we calculated the distance to a galaxy in the Virgo galaxy cluster using the brightest red giant stars in that galaxy. We measured those stars to have an ***apparent*** magnitude (in the I-band filter) of $m_I = 27.19 \pm 0.05$. If those stars have an ***absolute*** I-band magnitude given by $M_I = -4.05$, what is the distance and distance uncertainty to that galaxy? Give both values in megaparsecs.

Fun with Magnitudes! (10 points)

- A (very strange!) star cluster is made up of 10^6 stars identical to the Sun ($M_V = +4.82, B - V = 0.65$) and is at a distance of 10 kpc. What is its total V-band **absolute** magnitude? What is its total V-band **apparent** magnitude? What is its total $B - V$ color?

- Another very strange star cluster is made up of 10^6 Suns and another 10^5 red giants ($M_V = +1.00, B - V = 1.0$). What is its total V band absolute magnitude? What is its total $B - V$ color? What fraction of the total V band light of the cluster is coming from the red giants?

The Distance to The Stable[1] (10 points)

Here is [a dataset for an open cluster known as "The Stable"] (columns: apparent V mag and $B - V$ color). The Stable has a reddening of $\mathrm{E}(B - V) = 0.25$ magnitudes and roughly solar metallicity. Correct the colors and magnitudes for the dust (explain how you did this!) and the plot an observed color magnitude diagram (apparent mag vs color) for the Stable.

[1] The Stable is a real cluster in disguise: it's data for the Hyades star cluster, just shifted to a different distance.

Now, here is a a theoretical zero age main sequence (ZAMS) for individual stars with solar metallicity (Z=0.02) . The columns in the datafile are stellar mass (in solar masses), absolute V magnitude, and $B - V$ color.
Using that solar metallicity ZAMS, derive a distance to the Stable using main sequence fitting. Using your derived distance, convert the ZAMS *absolute* magnitudes to ZAMS *apparent* magnitudes ,and overplot the apparent magnitude ZAMS on your Stable data to show how good your match is.

Also estimate the error in your distance to the Stable, and explain what you think the main sources of error are.

Finally, what is the $B - V$ color of the stars at the main sequence turnoff? Use that turnoff color along with the figure below to estimate the age of this star cluster.

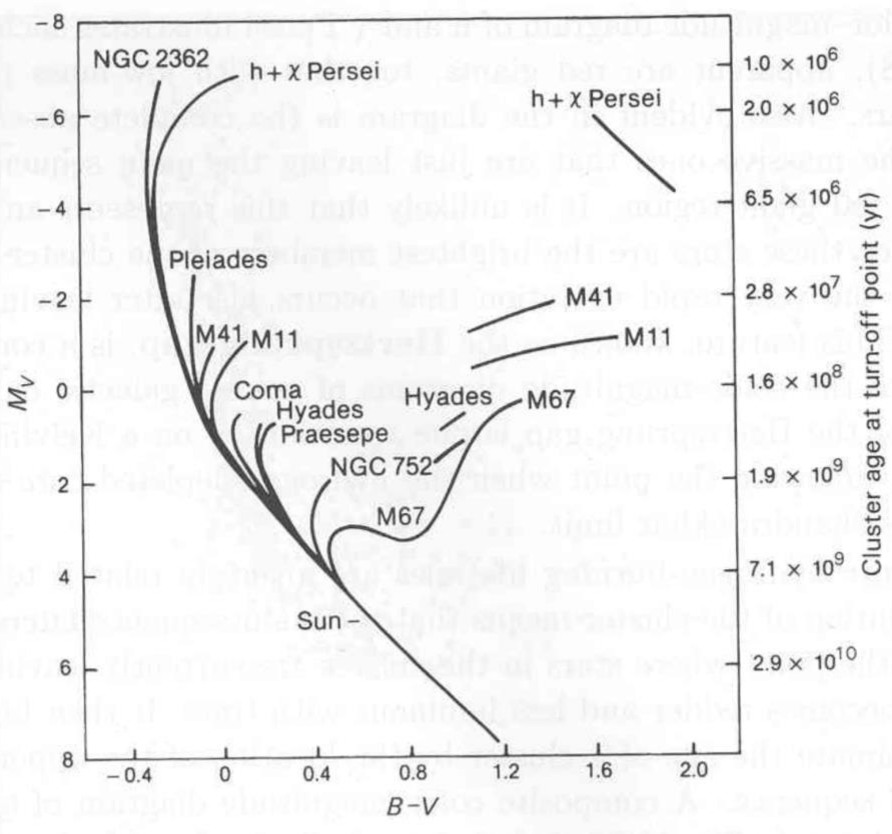

Figure 13.29 A composite color–magnitude diagram for a set of Population I galactic clusters. The absolute visual magnitude is indicated on the left-hand vertical axis and the age of the cluster, based on the location of its turn-off point, is labeled on the right-hand side. (Figure adapted from an original diagram by A. Sandage.)

The Distance to Laungheer 413[2] (10 points)

Here is photometry of Laungheer 413, a globular cluster. Again, you have apparent V magnitude and observed B-V color (note, however, that this dataset does not include red giant branch stars, only main sequence stars and stars just starting to evolve off the MS). Laungheer 413 has a reddening of $\mathrm{E}(B-V) = 0.1$ and a metallicity of $[\mathrm{Fe/H}] = -0.76$. Figure out the distance (and distance uncertainty) of Laungheer 413, the same way you did for the Stable. Since it is metal-poor, you'll want to compare it to this metal-poor (Z=0.004) ZAMS. Also use the main sequence turnoff point to get a rough age estimate, like you did for the Stable.

RR Lyrae Stars (10 points)

Laungheer 413 has one RR Lyrae variable star in it: V9, with a mean V apparent magnitude of 14.685 and period of 0.737 days. What is the mean absolute magnitude of V9 (remember to correct for the dust!)?

If you mistakenly thought it was a Cepheid, what would you have derived for its mean absolute magnitude given the Cepheid period-luminosity relationship? Under that (mistaken) assumption, what would you then estimate of the distance to Laungheer 413 to be?

(Use the calibrated period luminosity relationship $M_V = -2.43 \log P - 1.62$, taken from Benedict+07.)

[2] Similarly, Laungheer 413 is also a cluster in disguise, this time it's the globular cluster 47 Tuc, but again at a different distance.